On the localization-delocalization critical line for the random copolymer

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We consider the standard model for a random copolymer which has a Hamiltonian $H_n^{h,\beta,\omega}(\pi)$ on directed paths $\pi = \{(k,\pi_k) : k \in \mathbb{N}, \pi_k \in \mathbb{Z}\}$ given by

$$H_n^{h,\beta,\omega}(\pi) = -\beta \sum_{k=1}^n \left(\omega_k + h\right) \operatorname{sign}\left(\pi_k + \pi_{k-1}\right),$$

where the ω_k are i.i.d. real-valued centered random variables, for instance ± 1 , with probability 1/2. $h \ge 0$ and $\beta > 0$ are parameters. The Gibbs distribution is

$$P_{n}^{\beta,h,\omega}\left(\pi\right) = \frac{1}{Z_{n}^{h,\beta,\omega}} \exp\left[-H_{n}^{h,\beta,\omega}\left(\pi\right)\right] P\left(\pi\right)$$

with the a priory measure P on paths. A standard case is where P is the law of the ordinary nearest neighbor random walk.

The model has a nontrivial localization-delocalization transition: For $\beta > 0$, there is a critical value $h_c(\beta) > 0$ such that the path measure is localized for $h < h_c(\beta)$ and delocalized for $h > h_c(\beta)$, as has been proved in [2].

We present new results obtained with Frank den Hollander and A.A. Opoku on this critical line. In particular, a new lower bound for the tangent of the critical line at the origin is obtained. The results are proved by an application of a variational method developed by Birkner et al [1].

References

- Birkner, M., Greven, A., and den Hollander, F.: Quenched large deviation principle for words in a letter sequence. Prob. Th. Rel. Fields 148 403-456 (2010)
- [2] Bolthausen, E., and den Hollander, F.: Localization transition for a polymer near an interface. Ann. Prob. 25 1334-1366 (1997)