Traveling waves in a recurrently sawtoothed cylinder and their homogenization limit

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My talk is concerned with a curvature-dependent motion of plane curves in a two-dimensional cylinder with spatially undulating boundary. The law of motion is given by

$$V = \kappa + A,$$

where V is the normal velocity of the curve, κ is the curvature, and A is a positive constant. The boundary undulation is assumed to be recurrent. In other words, the boundary has many bumps that are aligned in a spatially recurrent manner. This includes periodic and quasi-periodic undulations as special cases.

We discuss how the average speed of the traveling wave depends on the geometry of the domain boundary. We will first give a necessary and sufficient condition for a traveling wave to exist. We then show that traveling waves have well-defined average speed if the undulation is uniquely ergodic.

Next we present an example of traveling wave whose average speed is zero. Such a peculiar situation, which we call "virtual pinning", can occur only in non-periodic environments.

We then consider the homogenization problem as the boundary undulation becomes finer and finer, and determine the homogenization limit of the average speed and the limit profile of the traveling waves. Quite surprisingly, this homogenized speed depends only on the maximal opening angles of the domain boundary and no other geometrical features are relevant. We also estimate the rate of convergence of the traveling wave speed to its homogenization limit. It turns out that this convergence rate is precisely $O(\sqrt{\varepsilon})$ when the boundary undulation is periodic, while it is slower in the aperiodic case, where ε is a parameter that represents the typical scale of the boundary undulation. In a more general case where the boundary undulation is quasi-periodic with m independent frequencies, our preliminary analysis suggests that the convergence rate is $O(\varepsilon^{2/(m+3)})$, with some logarithmic corrections. This formula shows that the convergence becomes slower as m increases. This is joint work with Bendong Lou and Ken-Ichi Nakamura.