Infinite Particle Systems associated with Airy kernel

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Let μ_{Ai} be the determinantal point process with the Airy kernel

$$K_{\mathrm{Ai}}(x,y) = \begin{cases} \dfrac{\mathrm{Ai}(x)\mathrm{Ai}'(y)-\mathrm{Ai}'(x)\mathrm{Ai}(y)}{x-y} & \text{if } x \neq y \\ (\mathrm{Ai}'(x))^2 - x(\mathrm{Ai}(x))^2 & \text{if } x = y, \end{cases} \tag{0.1}$$

where $\mathrm{Ai}(\cdot)$ is the Airy function defined by

$$\mathrm{Ai}(z) = \frac{1}{2\pi}\int_{\mathbb{R}} dk\, e^{i(zk+k^3/3)}, \quad z \in \mathbb{C},$$

and $\mathrm{Ai}'(x) = d\mathrm{Ai}(x)/dx$. In this talk we show that a diffusion process with reversible measure μ_{Ai} is constructed by Dirichlet form technique [2] and the process satisfies the following SDE:

$$dX_j(t) = dB_j(t) + \lim_{L\to\infty}\left\{ \sum_{k\neq j:|X_k(t)|<L} \frac{1}{X_j(t)-X_k(t)} - \int_{|x|<L} \frac{\rho(x)}{-x}dx, \right\} dt \quad j \in \mathbb{N},$$

where $B_j(t)$, $j \in \mathbb{N}$ are independent Brownian motions, $\rho(x) = K_{\mathrm{Ai}}(x,y)$, the density of μ_{Ai}, and the integral implies Cauchy principal value.

The infinite-dimensional determinantal process associated with the *extended Airy kernel* $\mathbf{K}_{\mathrm{Ai}}$:

$$\mathbf{K}_{\mathrm{Ai}}(s,x;t,y) \equiv \begin{cases} \displaystyle\int_0^\infty du\, e^{-u(t-s)/2}\mathrm{Ai}(u+x)\mathrm{Ai}(u+y) & \text{if } t \geq s \\ \displaystyle -\int_{-\infty}^0 du\, e^{-u(t-s)/2}\mathrm{Ai}(u+x)\mathrm{Ai}(u+y) & \text{if } t < s, \end{cases} \tag{0.2}$$

$x, y \in \mathbb{R}$, is also a Markov process with the reversible measure μ_{Ai} [1]. The coincidence of the above two processes has not been proved.

References

[1] Katori, M and Tanemura, H. : Markov property of determinantal processes with extended sine, Airy, and Bessel kernels, to appear in Markov process and related fields. arXiv:math.PR/11064360

[2] Osada, H. : Interacting Brownian motions in infinite dimensions with logarithmic interaction potentials. arXiv:math.PR/0902.3561

[3] Osada, H. : Infinite-dimensional stochastic differential equations related to random matrices, to appear in Probab. Theory Relat. Fields.