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On stochastic difference equations of parabolic type

We consider stochastic 'partial difference' equations associated with finite difference approximations in the space variable of stochastic parabolic partial differential equations (SPDEs), while the time variable changes continuously. Thus we study infinite systems of stochastic differential equations labelled by the grid points $\mathbb{G}_h$ for any grid $\mathbb{G}_h$ of mesh-size h compatible with the difference operators in the difference equations. Under appropriate parabolicity and regularity conditions we show that the solutions of these equations admit power series expansions in h with coefficients independent of h and remainder terms estimated independently of h. Thus Richardson's idea can be implemented to finite difference approximations of SPDEs. We show, in particular, that the order of accuracy of finite difference approximations for SPDEs can be made as high as we wish by taking appropriate mixtures of the approximations corresponding to different mesh-sizes, if the stochastic parabolicity condition is satisfied and the coefficients together with the initial data and free data satisfy appropriate regularity conditions. This result can be applied, for example, to finite difference approximations of the equations of nonlinear filtering of partially observed diffusion processes.
The talk is based on a recent joint work with N.V. Krylov.