Spatial random permutations and Bose-Einstein condensation

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Spatial random permutations differ from usual random permutations in that they are defined on a subset of a metric space; the probability of any given spatial random permutation then depends on the length and/or relative location of its jumps. A natural example is to assign an energy to each jump, which grows with the length of that jump; the total energy $H(\pi)$ of a permutation is then the sum of all its jump energies, and its probability is proportional to $\mathrm{e}^{-H(\pi)}$.

In the above situation, a phase transition is expected in the thermodynamic limit: for a high density of points, permutations should behave similarly to those without spatial constraints, and cycles of length proportional to the volume should emerge. For low densities of points, on the other hand, long cycles need to contain too many long jumps and become unlikely. Then it is expected that no infinite cycles appear.

In the annealed version of the model, where we average over the positions inside a large box, we prove that this phase transition indeed exists, and that infinite cycles appear above an explicit critical density in the thermodynamic limit. Moreover, the infinite cycles are distributed according to a Poisson-Dirichlet law.

I will give the main ideas needed to prove these results. Moreover, I will highlight the connection of spatial random permutations with the problem of Bose-Einstein condensation in the presence of interactions, at positive temperature. There, a series of approximations leads to a series of increasingly simpler models of spatial random permutations, where the last and simplest one can be treated rigorously with our results above. The approximations themselves are non-rigorous, although we expect them to be asymptotically accurate for small scattering length of the inter-particle potential. I will highlight the open problems in this context.