Simulated Annealing and Evolutionary Games

Yunshyong Chow *Institute of Mathematics, Academia Sinica, Taipei, Taiwan*
E-mail: chow@math.sinica.edu.tw

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Abstract: N players sit around a circle to play some game like the prisoner's dilemma game in which each player has only two strageties to play against his two neighbors. At time period $n = 1, 2, ...$, each player will first find, based on some criterion, a strategy that would benefilt him most in the next play, and then declares his final strategy by allowing to make a mistake independently with a small probability ϵ. The setup is like in interacting particle systems. It can be shown that $\mu_* \equiv \lim_{\epsilon \to 0} \mu_\epsilon$ exists, where μ_ϵ is the invariant measure as time goes to ∞ with ϵ fixed. For prisoner's dilemma games, the goal is find out whether the all-cooperation state is in the support of μ_* so that players could be out of the dilemma. For coordination games which has two Nash equilibria, the goal is to find out which one is in the support of μ_* and thus stands out in the long run.

The mutation mechanism is vital in biological evolutions. Here, it allows us to use results from simulated annealing to obtain some interesting results.

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