Rigorous exponents for four dimensional random walk trace

Daisuke Shiraishi

Department of Mathematics
Faculty of Science
Kyoto University

daisuke@math.kyoto-u.ac.jp

Abstract

Let S be a simple random walk starting at the origin in $\mathbb{Z}^4$. We consider $\mathcal{G} = S[0, \infty)$ to be a random subgraph of the integer lattice and assume that a resistance of unit 1 is put on each edge of the graph $\mathcal{G}$. Let R_n be the effective resistance between the origin and S_n. We derive the exact value of the resistance exponent; more precisely, we prove that $n^{-1}E(R_n) \approx (\log n)^{-\frac{1}{2}}$. We also show that the chemical exponent is equal to the resistance exponent; namely, we prove that $n^{-1}E(d(0, S_n)) \approx (\log n)^{-\frac{1}{2}}$, where $d(\cdot, \cdot)$ is the graph distance on $\mathcal{G}$. Furthermore, we derive the precise exponent for the heat kernel and the mean-square displacement of a random walk on $\mathcal{G}$ at the quenched level. These results give the answer to the problem raised by Burdzy and Lawler (1990) in four dimensions.

References

[1] Burdzy, K.; Lawler, G. F. : Rigorous exponent inequalities for random walks. Journal of Physics A: Mathematical and General, Volume 23, Issue 1, pp. L23-L28 (1990).