Complex Brownian Motion Representation of the Dyson model

Hideki Tanemura, Chiba University
(*joint work with Makoto Katori, Chuo University*)

Dyson's Brownian motion model with the parameter $\beta = 2$, which we simply call the Dyson model in this talk, is realized as an h-transform of the absorbing Brownian motion in a Weyl chamber of type A. Depending on initial configuration of the Dyson model with a finite number of particles, we define a set of entire functions and introduce a martingale for a system of independent complex Brownian motions (CBMs), which is expressed by a determinant of a matrix with elements given by the conformal transformations of CBMs by the entire functions. We prove that the Dyson model can be represented by the system of independent CBMs weighted by this determinantal martingale. From this CBM representation, the Eynard-Mehta-type correlation kernel is derived and the Dyson model is shown to be determinantal. The CBM representation is a useful extension of h-transform, since it works also for infinite particle systems. Using this representation, we prove the tightness of a series of processes, which converges to the Dyson model with an infinite number of particles.

References

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