Stochastic Contact Geometry and associated time reversible Dynamics

Jean-Claude Zambrini, GFMUL, Lisbon, Portugal

We advocate a dynamical system approach to Stochastic Analysis, where classical deterministic systems are deformed stochastically,in a way preserving most of their geometrical structures. Our starting points are Expectations of Action functionals, typical of Stochastic Control Theory but their Lagrangians are now taken seriously in order to construct dynamical theories. It is shown that Hamilton-Jacobi-Bellman equations, in particular, can be regarded as stochastic deformations of their classical counterparts and that their geometrical structure is contained in a deformation of Cartan's method of Ideal of differential forms. Stochastic versions of Noether's Theorem provide martingales of the Markovian processes which are extremals of the Actions. This should allow to define a general notion of integrability for such stochastic dynamical systems.

The notion of reversibility,for those extremal processes,is analyzed. It is shown that they correspond, in fact, to a deformation of the solutions of boundary value classical variational problems. The underlying class of reversible processes is, indeed, more general than Markovian, and their construction quite different from the traditional ones (inspired by initial or final value problems).

A deformation of the classical Maupertui's principle provides a notion of "Integration by quadrature",the simplest example of stochastic integrability.

This method of stochastic deformation is very general ,and compatible with different kind of stochastic processes.