Lower bound estimate of the spectral gap for simple exclusion process with degenerate rates.

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Simple exclusion process with degenerate rates is one of the simplest system called *kinetically constrained lattice gases*, which have been introduced in the physical literature as simplified models for some peculiar phenomena of the "glassy" dynamics.

Let us consider discrete torus $T_n = \{1, 2, \dots n\}$ (n is identified with 0). We define the set of configurations by $\Sigma_n := \{0, 1\}^{T_n}$, the set of configurations conditioned by the number of particles by $\Sigma_{n,k} := \{\eta \in \Sigma_n; \sum_{x \in T_n} \eta_x = k\}$.

For $\eta \in \Sigma_n$ and $x, y \in T_n$, we define the configuration $\eta^{x,y} \in \Sigma_n$ by $(\eta^{x,y})_x = \eta_y$, $(\eta^{x,y})_y = \eta_x$, and $(\eta^{x,y})_z = \eta_z$ for $z \neq x, y$, and the operator $\pi^{x,y}$ by $\pi^{x,y} f(\eta) = f(\eta^{x,y}) - f(\eta)$. We set $c(\eta) := \eta_{-1} + \eta_2$. Let τ_x be a shift operator. Given a local function g, which is strictly positive and does not depend on the value of η_0 nor η_1, we define the generator of simple exclusion process with degenerate rate $L = L_g$ by

$$Lf(\eta) = \sum_{x \in T_n} \tau_x(c(\eta)g(\eta))\pi^{x,x+1} f(\eta)$$

for all local function f.

The ergodic component of the system is complicated. If the density of the particle is large, precisely if $k > n/3$, then $\Sigma_{n,k}$ is an ergodic component. If the density of the particle is small, precisely if $k \leq n/3$, then $\Sigma_{n,k}$ is decomposed to blocked configurations and a component which contains all configurations with at least one couple of particles at distance at most two. We define

$$\Sigma^0_{n,k} := \{\eta \in \Sigma_{n,k}; \sum_{x \in T_n} (\eta_x \eta_{x+1} + \eta_x \eta_{x+2}) > 0\},$$
$$\Sigma^1_{n,k} := \{\eta \in \Sigma_{n,k}; \sum_{x \in T_n} (\eta_x \eta_{x+1} + \eta_x \eta_{x+2}) = 0\}.$$

Note that $\Sigma^1_{n,k}$ is set of all blocked configurations. We also note that if $k > n/3$ then $\Sigma^0_{n,k} = \Sigma_{n,k}$. It is not difficult to see that $\Sigma^0_{n,k}$ is an ergodic component of the system. Let $\mu = \mu_{n,k}$ be a uniform probability measure on $\Sigma^0_{n,k}$. Then it is easy to see that L is reversible with respect to μ. Let $L_{n,k}$ be the restriction of L on $\Sigma^0_{n,k}$. Then we can consider the spectral gap of $-L_{n,k}$, which is defined by

$$\lambda = \lambda(n,k) := \inf \left\{ \frac{E[f(-L_{n,k})f]}{E[f^2]} \middle| E[f] = 0 \right\}.$$

Theorem 1 There exists a constant C not depending on n nor k such that

$$\lambda(n,k) \geq C \frac{\rho^4}{n^2}$$

where $\rho = k/n$.