Hydrodynamic limit for the Ginzburg-Landau $\nabla\phi$ interface model
with both a conservation law and the Dirichlet boundary condition

Takao Nishikawa (Nihon University)

We discuss the hydrodynamic scaling limit for the interface motion preserving its volume. We consider the dynamics governed by SDEs

$$d\phi_t(x) = -(-\Delta)\frac{\partial H}{\partial \phi(\cdot)}(\phi_t)dt + \sqrt{-2\Delta}w_t(x),$$

where Δ is the discrete Laplacian and H is the Hamiltonian defined by

$$H(\phi) = \sum_{x\sim y} V(\phi(x) - \phi(x)).$$

In this setting, the macroscopic equation is identified in [N. 2002] for the dynamics on the d-dimensional torus. Precisely saying, via the scaling by the factor N^{-1} for space and the factor N^4 for time, the nonlinear fourth-order partial differential equation

$$\frac{\partial h}{\partial t} = -\Delta \mathrm{div}\{(\nabla\sigma)(\nabla h)\}$$

appears as the limit equation of scaled height variables, where $\sigma : \mathbb{R}^d \to \mathbb{R}$ is the function so called "surface tension." The aim of this talk is to discuss the behavior of the interface motion on a finite domain with both the conservation law and the Dirichlet boundary condition, and to derive the macroscopic equation. We also discuss the relationship with the Wulff shape derived by [Deuschel-Giacommin-Ioffe, 2001].