A NEW VIEW ON LACE EXPANSIONS AND SELF-AVOIDING RANDOM WALKS

ERWIN BOLTHAUSEN

The lace expansion was introduced in a seminal paper by Brydges and Spencer to prove the diffusive behavior of (weakly) self-avoiding random walks in dimensions above 4. The expansion is quite simple, at least for the SAW, but there is considerable work needed to derive from it the diffusive behavior. We present some recent developments using contraction techniques directly in x-space. This leads to general results for solutions of convolution equations of which the one coming from the SAW is just a special case. As applications, it leads for the SAW to new results for walks in continuous space, asymmetric SAWs, and Green's functions, and for the classical situation, it leads to local results which are sharper than those obtained by other methods.

This is joint work with Christine Ritzmann, Luca Avena, Felix Rubin, and work in progress with Remco van der Hofstad, and Gady Kozma.

A CENTRAL LIMIT THEOREM FOR THE EFFECTIVE CONDUCTANCE

MAREK BISKUP

As is well known, most metals, regardless how pristine they may seem at the macroscopic scale, have a rather complicated microscopic structure. This naturally leads to the question why their conductivity properties are governed by equations with smooth coefficients. An answer is offered by homogenization theory: smoothness arises via self-averaging.

In my talk I will consider a specific problem of electric conductivity of random resistor networks on $\mathbb{Z}^d$. To each unordered pair $\langle x, y\rangle$ of nearest-neighbor vertices we assign a resistance $r_{xy} = r_{yx}$ or, alternatively, a conductance $c_{xy} = 1/r_{xy}$. For a finite connected set $V \subset \mathbb{Z}^d$, with incident edges, resp., outer boundary vertices denoted by $\mathbb{B}(V)$, resp., ∂V, we are looking for the minimum of the Dirichlet energy

$$Q_V(f) := \sum_{\langle x,y\rangle\in\mathbb{B}(V)} c_{xy}\bigl[f(y) - f(x)\bigr]^2$$

over all $f\colon V \cup \partial V \to \mathbb{R}$ such that $f = g$ on ∂V. In the specific case of a square box $V_L := (0, L)^d \cap \mathbb{Z}^d$ and linearly-growing boundary conditions, $g(x) := t \cdot x$, we define the effective conductance by

$$C_L^{\text{eff}}(t) := \inf\bigl\{Q_{V_L}(f)\colon f(x) = t \cdot x,\ \forall x \in \partial V_L\bigr\}$$

Assuming that the conductances c_{xy} are independent and identically distributed, we aim to describe the asymptotic distributional properties of $C_L^{\text{eff}}(t)$ when L is very large.

It is well known that, under mild integrability conditions on c_{xy}, the quantity $C_L^{\text{eff}}(t)/L^d$ scales to a deterministic constant. Assuming a small ellipticity contrast, i.e., the existence of a small-enough $\lambda > 1$ such that

$$\frac{1}{\lambda} \le c_{xy} \le \lambda,$$

I will outline the proof of a central limit theorem,

$$\frac{C_L^{\text{eff}}(t) - \mathbb{E}C_L^{\text{eff}}(t)}{L^{d/2}} \underset{L\to\infty}{\overset{\text{law}}{\longrightarrow}} \mathcal{N}(0, \sigma_t^2),$$

for some σ_t^2 which is positive whenever $t \neq 0$ and is, in fact, of a bi-quadratic form in t. Among the key ingredients are the corrector method from homogenization theory and the Martingale Central Limit Theorem; an integrability condition is furnished by the Meyers estimate.

The talk is based on a joint paper with M. Salvi and T. Wolff. Time permitting, I will comment on related results due to P. Nolen and, independently, R. Rosignol that deal with periodic boundary conditions, and a method to remove the restriction on the ellipticity contrast.

Effective resistances for supercritical percolation clusters in boxes

Yoshihiro Abe (Kyoto University)

Consider the largest supercritical percolation cluster in $[-n, n]^d \cap \mathbb{Z}^d, d \geq 2$. In this talk, I will describe an estimate on effective point-to-point resistances for the largest cluster. The estimate implies that the cover time for the simple random walk on the largest cluster is comparable to $n^d (\log n)^2$. It is well known that the cover time for the simple random walk on $[-n, n]^d \cap \mathbb{Z}^d$ is of order $n^d \log n$ for $d \geq 3$ and of order $n^2 (\log n)^2$ for $d = 2$. The result exhibits a quantitative difference between the two walks for $d \geq 3$. The proof is based on classical percolation techniques such as Kesten's crossing probability estimate and the static renormalization argument.

ON HYDRODYNAMIC LIMIT FOR SIMPLE EXCLUSION PROCESS WITH DEGENERATE RATES

YUKIO NAGAHATA

Simple exclusion process with degenerate rates is one of the simplest system called *kinetically constrained lattice gases*, which have been introduced in the physical literature as simplified models for some peculiar phenomena of the "glassy" dynamics.

Let us consider discrete torus $T_n = \{1, 2, \ldots n\}$ (n is identified with 0). We define the set of configurations by $\Sigma_n := \{0,1\}^{T_n}$, the set of configurations conditioned by the number of particles by $\Sigma_{n,k} := \{\eta \in \Sigma_n; \sum_{x \in T_n} \eta_x = k\}$.

For $\eta \in \Sigma_n$ and $x, y \in T_n$, we define the configuration $\eta^{x,y} \in \Sigma_n$ by $(\eta^{x,y})_x = \eta_y$, $(\eta^{x,y})_y = \eta_x$, and $(\eta^{x,y})_z = \eta_z$ for $z \neq x, y$, and the operator $\pi^{x,y}$ by $\pi^{x,y} f(\eta) = f(\eta^{x,y}) - f(\eta)$. We set $c(\eta) := \eta_{-1} + \eta_2$. Let τ_x be a shift operator. Given a local function g, which is strictly positive and does not depend on the value of η_0 nor η_1, we define the generator of simple exclusion process with degenerate rate $L = L_g$ by

$$Lf(\eta) = \sum_{x \in T_n} \tau_x(c(\eta)g(\eta))\pi^{x,x+1} f(\eta)$$

for all local function f.

This model is "non-gradient" type. To establish the hydrodynamic limit for non-gradient model, "gradient replacement" lemma plays a key role. I will talk on "gradient replacement" lemma for this model.

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Growth exponent for loop-erased random walk in three dimensions

Daisuke Shiraishi

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Abstract

Let S be the simple random walk on $\mathbb{Z}^3$ started at the origin and σ_n be the first time S exits the ball of radius n with center the origin. Let M_n be the number of steps of $\mathrm{LE}(S[0,\sigma_n])$, the loop-erasure of $S[0,\sigma_n]$. Physicists conjecture that there exists α (called the growth exponent) such that $E(M_n) \approx n^{\alpha}$ and did numerical experiments to show that $\alpha = 1.62 \pm 0.01$ ([1]). However, rigorously the existence of α is not proved.

In the talk, I show the existence of the growth exponent, i.e., I prove that there exists α such that

$$\lim_{n\to\infty} \frac{\log E(M_n)}{\log n} = \alpha.$$

References

[1] A. J. Guttmann and R. J. Bursill, Critical exponents for the loop erased self-avoiding walk by Monte Carlo methods, Journal of Statistical Physics 59:1/2 (1990), 1-9.

ON THE CONFORMAL STRUCTURE OF RANDOM TRIANGULATIONS

NICOLAS CURIEN

A (planar) triangulation is a graph embedded in the two-dimensional sphere such that all its faces are surrounded by three edges. Consider a random triangulation T_n chosen uniformly over all triangulations of the sphere having n faces. The metric structure of T_n endowed with the graph distance has been studied in depth during recent years. In particular, Le Gall and Miermont recently proved that the metric space obtained from T_n by re-scaling all distances by $n^{-1/4}$ converges towards a random compact metric space called "the Brownian map".

In this talk, we will focus on another aspect of random triangulations. Indeed, T_n can naturally be considered as a random Riemann surface and one can study its "conformal structure" which is conjectured to be strongly linked to the Gaussian free field. I will present a path to study the conformal structure of random planar maps based on their Markovian exploration by an independent SLE_6 process.

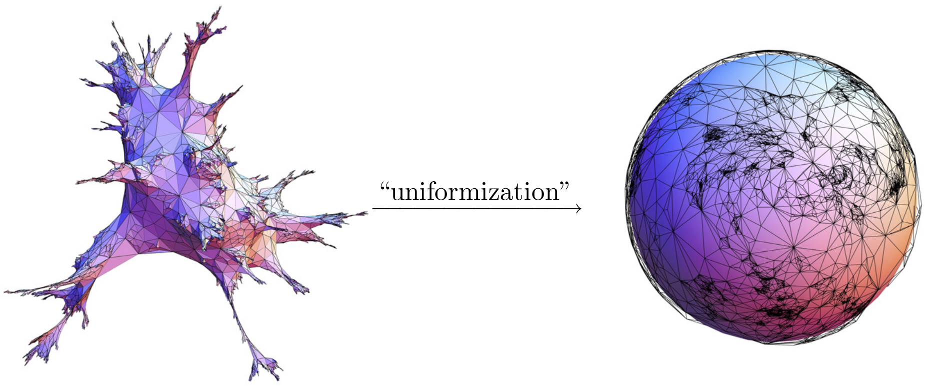

FIGURE 1. A random triangulation embedded not isometrically in $\mathbb{R}^3$ and an approximation of its uniformization on the two-dimensional sphere.

PERSISTENCE PROBABILITY FOR A CLASS OF GAUSSIAN PROCESSES RELATED TO RANDOM INTERFACE MODELS

HIRONOBU SAKAGAWA

We consider a class of Gaussian processes which are obtained as height processes of some $(d+1)$-dimensional dynamic random interface model on $\mathbb{Z}^d$. We give an estimate of persistence probability, namely, large T asymptotics of the probability that the process does not exceed a fixed level up to time T. The interaction of the model affects the persistence probability and its asymptotic behavior changes depending on the dimension d.

RIGIDITY PHENOMENA IN RANDOM POINT SETS AND APPLICATIONS

SUBHRO GHOSH

In several naturally occurring (infinite) point processes, we show that the number (and other statistical properties) of the points inside a finite domain are determined, almost surely, by the point configuration outside the domain. This curious phenomenon we refer to as "rigidity". We will discuss rigidity phenomena in point processes and their applications. Depending on time, we will talk about applications to stochastic geometry and to random instances of some classical questions in Fourier analysis.

Princeton University.

THE DYADIC MODEL WITH NOISE: UNIQUENESS, EXPLOSION AND ANOMALOUS DISSIPATION

MARCO ROMITO

The talk is a presentation and a review of the dyadic model of turbulence. We shall derive the model from fluid dynamics equations and present its main features: uniqueness, explosion and anomalous dissipation, in dependence of the strength of the interaction, of the presence of a friction and of the input noise.

ENERGY DIFFUSION FOR SOME STOCHASTIC PARTICLE SYSTEMS WITH MECHANICAL ORIGIN

MAKIKO SASADA (KEIO UNIVERSITY)

To derive Fourier's law from mechanical models, we consider N-particle stochastic systems obtained as mesoscopic dynamics for energy transfer from N-particle microscopic mechanical models. From locally confined particles in interaction, and weakly coupled geodesic flows on d-dimensional manifolds of negative curvature, we obtain pure jump processes, called stochastic energy exchange models, and diffusion processes, called conservative energy Ginzburg-Landau models, describing the dynamics of energy distribution, respectively. We extract important common properties between them and characterize macroscopic behaviors of their generalizations under the condition that these common properties hold. More precisely, we give some results on the reversible measure, the spectral gap estimate and the macroscopic diffusion coefficient for these generalized processes under "natural" conditions.

GEOMETRIC AND DYNAMICAL RIGIDITY OF STOCHASTIC COULOMB SYSTEMS IN INFINITE-DIMENSIONS

HIROFUMI OSADA

Stochastic Coulomb dynamics in infinite-dimensions are infinitely many Brownian particles in $\mathbb{R}^d$ interacting via γ-dimensional Coulomb potential Ψ_γ with inverse temperature β. When the systems are translation invariant, then the dynamics are given by the infinite-dimensional stochastic differential equations

$$dX_t^i = dB_t^i - \frac{\beta}{2} \lim_{r\to\infty} \sum_{j\neq i,\, |X_t^i - X_t^j|<r} \nabla\Psi_\gamma(X_t^i - X_t^j)dt \tag{1}$$

We suppose $d \le \gamma \le d+2$ because Ψ_γ become Ruelle class potentials if $\gamma > d+2$, and it seems difficult to justify the SDEs (1) for $\gamma < d$. If $d \le \gamma \le d+2$, then we call (1) (translation invariant) stochastic Coulomb dynamics. When $\gamma = d$, we call (1) *strict* stochastic Coulomb dynamics. So far the only example of strict stochastic Coulomb dynamics is the Ginibre interacting Brownian motions, namely the case $(\beta, \gamma, d) = (2, 2, 2)$. Dysons models and Airy Interacting Brownian motions are examples of stochastic Coulomb dynamics in one dimension with two-dimensional Coulomb potentials. Namely, $(\beta, \gamma, d) = (1, \gamma, d), (2, \gamma, d), (4, \gamma, d)$. (see [2], [3], [4], [5], [9], [10]).

In this talk, I present various dynamical rigidity of the Ginibre interacting Brownian motions. In particular, I prove that the tagged particles of Ginibre interacting Brownian motions are <u>sub-diffusive</u> [8].

Such a sub-diffusivity of tagged particles may be surprising. For the tagged particles of interacting Brownian motions in $\mathbb{R}^d$ with $d \ge 2$ with Ruelle's class potentials are always diffusive. This diffusivity is proved in [1] for Ruelle's class potentials with convex hard cores. In case of no hard core, this has been not yet fully proved, but generally believed by the specialists in mathematics, and papers in physics are treated as a fact.

In addition, I give a phase transition conjecture on the rigidity of strict stochastic Coulomb dynamics with the inverse temperature β for general dimensions $d \ge 2$.

The proof is based on the results of stochastic geometry developed in [6] and [7].

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[6] Osada, H., Shirai, T., *Absolute continuity and singularity of Palm measures of the Ginibre point process*, (preprint/draft)

Date: 2013/11/21–23.

Kyushu University.

[7] Osada, H., *Palm decomposition and restore density formulae of the Ginibre point process*, (in preparation)
[8] Osada, H., *Sub-diffusivity of tagged particles of Ginibre interacting Brownian motions*, (in preparation)
[9] Osada, H., Tanemura, H., *Infinite-dimensional stochastic differential equations arising from Airy random point fields*, (preprint/draft)
[10] Osada, H., Tanemura, H., *Strong solutions of infinite-dimensional stochastic differential equations and tail theorems*, (preprint/draft)

DETERMINANTAL MARTINGALES AND CORRELATIONS OF NONCOLLIDING RANDOM WALKS

MAKOTO KATORI

We study the noncolliding random walk (RW), which is a particle system of one-dimensional, simple and symmetric RWs starting from distinct even sites and conditioned never to collide with each other. When the number of particles is finite, $N < \infty$, this discrete process is constructed as an h-transform of absorbing RW in the N-dimensional Weyl chamber. We consider Fujita's polynomial martingales of RW with time-dependent coefficients and express them by introducing a complex Markov process. It is a complexification of RW, in which independent increments of its imaginary part are in the hyperbolic secant distribution, and it gives a discrete-time conformal martingale. The h-transform is represented by a determinant of the matrix, whose entries are all polynomial martingales. From this determinantal-martingale representation (DMR) of the process, we prove that the noncolliding RW is determinantal for any initial configuration with $N < \infty$, and determine the correlation kernel as a function of initial configuration. We show that noncolliding RWs started at infinite-particle configurations having equidistant spacing are well-defined as determinantal processes and give DMRs for them. Tracing the relaxation phenomena shown by these infinite-particle systems, we obtain a family of equilibrium processes parameterized by particle density, which are determinantal with the discrete analogues of the extended sine-kernel of Dyson's Brownian motion model with $\beta = 2$. Following Donsker's invariance principle, convergence of noncolliding RWs to the Dyson model is also discussed.

Chuo University, Tokyo.

Strong Markov property of determinatal processes associated with extended kernels

Hideki Tanemura, Chiba University

We denote by $\mathfrak{M}$ the space of nonnegative integer-valued Radon measures on $\mathbb{R}$, which is a Polish space with the vague topology. Any element ξ of $\mathfrak{M}$ can be represented as $\xi(\cdot) = \sum_{j\in\Lambda} \delta_{x_j}(\cdot)$ with a sequence of points in $\mathbb{R}$, $\boldsymbol{x} = (x_j)_{j\in\Lambda}$ satisfying $\xi(K) = \sharp\{x_j : x_j \in K\} < \infty$ for any compact subset $K \subset \mathbb{R}$. The index set $\Lambda = \mathbb{N} \equiv \{1, 2, \dots\}$ or a finite set. We call an element ξ of $\mathfrak{M}$ an unlabeled configuration, and a sequence $\boldsymbol{x}$ a labeled configuration. As a generalization of a notion of determinantal point process on $\mathbb{R}$ for a probability measure on $\mathfrak{M}$, we give the following definition for $\mathfrak{M}$-valued processes.

Definition 1 *An $\mathfrak{M}$-valued process $(\mathbb{P}, \Xi(t), t \in [0,\infty))$ is said to be determinantal with the correlation kernel $\mathbb{K}$, if for any $M \quad 1$, any sequence $(N_m)_{m=1}^{M}$ of positive integers, any time sequence $0 < t_1 < \cdots < t_M < \infty$, the $(N_1, \dots, N_M)$-multitime correlation function is given by a determinant,*

$$\rho\Big(t_1, \xi^{(1)}; \dots; t_M, \xi^{(M)}\Big) = \det_{\substack{1\le j\le N_m, 1\le k\le N_n \\ 1\le m,n\le M}} \Big[\mathbb{K}(t_m, x_j^{(m)}; t_n, x_k^{(n)})\Big],$$

where $\xi^{(m)}(\cdot) = \sum_{j=1}^{N_m} \delta_{x_j^{(m)}}(\cdot), 1 \le m \le M$.

We consider the determinatal processes $(\mathbb{P}^{\xi}, \Xi(t))$ associated with the extended sine kernel, extended Airy kernel and extended Bessel kernel. These processes are reversible Markov process [1]. In this talk we discuss the following:

1. The Strong Markov property of the processes
2. The SDEs and Dirichlet forms related to the processes

References

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Remarks on α-determinantal point fields

Tomoyuki SHIRAI

Determinantal point processes are point processes with determinantal correalation functions. Permanental point processes are also known to be those with permanental correlation functions. In [1], we tried to interpolate these point processes by introducting 1-parameter α in their Laplace transforms. In the study, we encounter a positivity problem of the so-called α-determinant (slightly different version is also called α-permanent), which is a generalization of determinant and permanent. This problem is closely related to the existence problem of α-determinatal point process. We briefly review this problem and related known results. Also, we discuss a special α-determinantal point process associated with green kernel and give some remarks on relationship between occupation fields of a loop space.

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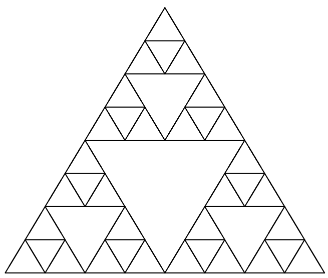
Random spanning trees on Sierpiński gasket graphs

Masato SHINODA

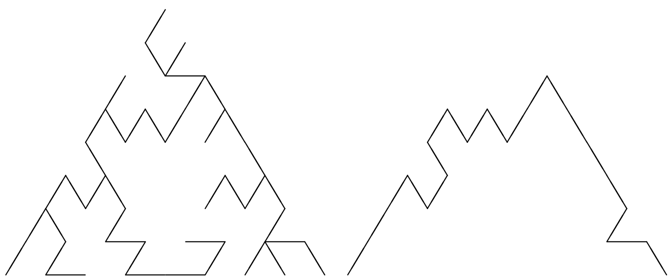
We study spanning trees on Sierpiński gasket graphs (i.e. finite approximations to the Sierpiński gasket). Let G_n be the nth Sierpiński gasket graph and let $\mathcal{T}_n$ be the set of the spanning trees of G_n. A spanning tree $\omega \in \mathcal{T}_n$ induces a self-avoiding walk from one corner of G_n to another (See the figure below).

There are two important probabilistic models of random spanning trees: (i) *uniform spanning tree* (UST) and (ii) *minimal spanning tree* (MST). For these models we show some geometric properties of spanning trees and induced self-avoiding walks.

[1] Shinoda,M., Teufl,E. and Wagner,S. Uniform spanning trees on Sierpinski graphs, arXiv:1305.5114.



G_3



a spanning tree of G_3 and induced self-avoiding walk

Experimental evidence for universal fluctuation properties of growing interfaces

Kazumasa A. Takeuchi
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I will present recent developments on growing interfaces showing universality beyond the scaling exponents, along an experimental realization found in chaotic regimes of electrically-driven liquid crystal convection. Measuring interface fluctuations of growing domains (photos below), we found not only the scaling exponents of the Kardar-Parisi-Zhang (KPZ) universality class, but also particular distribution and correlation functions that were previously derived for solvable models in the KPZ class (figure). Interestingly, these statistical properties have direct yet non-trivial link to random matrix theory, depend on the global geometry of the interfaces (whether the interfaces are curved or not), but are nevertheless universal in each case (see figure). In other words, the KPZ class splits into a few universality subclasses. These results constitute direct evidence for powerful universality of the KPZ class, ruling detailed statistical properties like distribution and correlation functions.

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K. A. Takeuchi and M. Sano, Phys. Rev. Lett. 104, 230601 (2010); J. Stat. Phys. 147, 853 (2012).

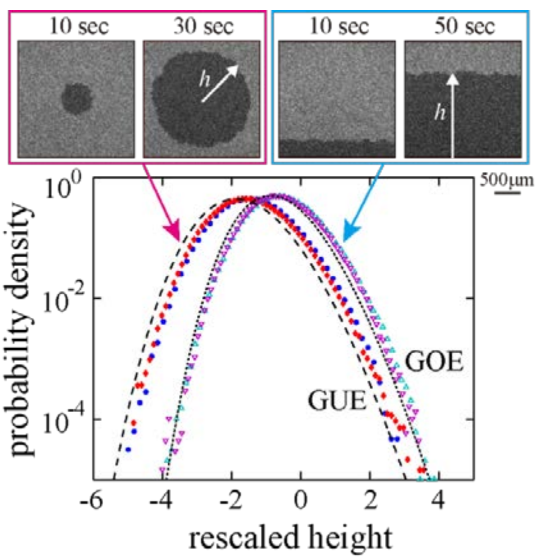

Figure: Growing domains of chaotic liquid-crystal convection. Fluctuations of the height h (as defined in the top figures) for the circular and flat interfaces obey the largest-eigenvalue distributions of the GUE and GOE random matrices, respectively, in the appropriately rescaled units.

Fluctuations for one-dimensional Brownian motions with oblique reflection

T. Sasamoto, Chiba University

We consider a system of Brownian motions in one-dimension in which the jth particle is reflected by the $(j+1)$th particle with weight p and also by the $(j-1)$th particle with weight q, where $j \in \mathbb{N}$ and $p \geq 0, q \geq 0, p+q=1$. More precisely, for the case with m particles, we consider $y(t) = (y_1(t), \ldots, y_m(t))$ with $y_1(t) \leq \ldots \leq y_m(t)$ which satisfies

$$y_j(t) = y_j + B_j(t) - p\Lambda^{(j,j+1)}(t) + q\Lambda^{(j-1,j)}(t) \tag{0.1}$$

where $\Lambda^{(0,1)}(t) = \Lambda^{(m,m+1)}(t) = 0$ and

$$\Lambda^{(j,j+1)}(\cdot) = L^{y_{j+1}-y_j}(\cdot, 0) \tag{0.2}$$

is the local time for $y_{j+1}(\cdot) - y_j(\cdot)$. The system with a symmetric ($p = q = 1/2$) reflection, corresponding to independent Brownian motions with ordering maintained, was introduced by Harris in 1965 and has been studied by many authors since then.

In the totally asymmetric ($q = 1$) case, a particle with smaller index has a priority to the one with a larger index; the latter is simply reflected by the former. The finite particle system for this special case was discussed by Warren and others. More recently, there have been some progress for more general initial conditions by by Ferrai, Spohn and Weiss.

In this presentation we consider generic asymmetric case where $0 < p < q < 1$. The large time properties are expected to be similar to the totally asymmetric case, i.e., to belong to the KPZ universality class. But the techniques for the totally asymmetric case do not work for the general case. Our analysis is based on a duality (in fact a self-duality) property for the process, which allows us to obtain a few formulas for quantities related to current and discuss the asymptotics. A similar analysis has been done for the case of the asymmetric simple exclusion process(ASEP) in [BCS]. The main purpose of the presentation is to explain the applicability of the methods to the Brownian motion case and demonstrate its usefulness.

The presentation is based on a collaboration with H. Spohn.

[BCS] A. Borodin, I. Corwin, T. Sasamoto, From duality to determinants for q-TASEP and ASEP, arXiv:1207.5035, to appear in Ann. Prob.

KPZ equation, its renormalization and invariant measures

T. Funaki (Univ. Tokyo)

The Kardar-Parisi-Zhang (KPZ) equation is a stochastic partial differential equation of the form

$$\partial_t h = \frac{1}{2}\partial_x^2 h + \frac{1}{2}\big(\partial_x h\big)^2 + \dot{W}(t,x), \quad x \in \mathbb{R},$$

where $\dot{W}(t,x)$ is the space-time Gaussian white noise, which has the correlation function

$$E[\dot{W}(t,x)\dot{W}(s,y)] = \delta(x-y)\delta(t-s).$$

We consider in one dimension on the whole line. This equation is ill-posed because of inconsistency between the nonlinearity and the roughness of the noise. However, its Cole-Hopf solution defined as the logarithm of the solution of the linear stochastic heat equation (SHE) with a multiplicative noise:

$$\partial_t Z = \frac{1}{2}\partial_x^2 Z + Z\dot{W}(t,x), \quad x \in \mathbb{R},$$

i.e., $h(t,x) := \log Z(t,x)$ is a mathematically well-defined object and, in fact, M. Hairer has recently proved that the solution of SHE can be derived through the Cole-Hopf transform of the solution of the KPZ equation with a suitably renormalized factor under the periodic boundary condition.

In this talk, we introduce a different type of renormalization for the KPZ equation on the whole line:

$$\partial_t h = \frac{1}{2}\partial_x^2 h + \frac{1}{2}\big((\partial_x h)^2 - \xi^\varepsilon\big) * \eta_2^\varepsilon + \dot{W}^\varepsilon(t,x), \quad x \in \mathbb{R},$$

where a smeared noise $\dot{W}^\varepsilon(t,x) = \dot{W} * \eta^\varepsilon(t,x)$, $\eta_2^\varepsilon = \eta_\varepsilon * \eta_\varepsilon$ and $\xi^\varepsilon = \eta_2^\varepsilon(0)$ are defined from a usual convolution kernel η^ε which tends to δ_0 as $\varepsilon \downarrow 0$. This type of renormalization is appropriate from the view point to characterize the invariant measures. The Cole-Hopf transform applied to this equation leads to an SHE with a smeared noise having an extra complex nonlinear term involving a certain renormalization structure:

$$\partial_t Z = \frac{1}{2}\partial_x^2 Z + \frac{1}{2}Z\left\{\left(\frac{\partial_x Z}{Z}\right)^2 * \eta_2^\varepsilon - \left(\frac{\partial_x Z}{Z}\right)^2\right\} + Z\dot{W}^\varepsilon(t,x), \quad x \in \mathbb{R}.$$

It is shown that, under the situation that the corresponding tilt process is stationary, this complex term (the middle term in the RHS) can be replaced by a simple linear term divided by a specific constant 24 in the limit, so that the limit equation is the linear SHE:

$$\partial_t Z = \frac{1}{2}\partial_x^2 Z + \frac{1}{24}Z + Z\dot{W}(t,x), \quad x \in \mathbb{R}.$$

The Wiener-Itô expansion and a similar method for establishing the so-called Boltzmann-Gibbs principle are effectively used. As a result, it is shown that the distribution of a two-sided geometric Brownian motion with a height shift given by Lebesgue measure is invariant under the evolution determined by the SHE on $\mathbb{R}$.

Multi-component KPZ equation will be also discussed at approximating level and we study its invariant measures.

This is a joint work with Jeremy Quastel.

Large deviations for random walks with random holding times

Ryoki Fukushima
(Research Institute for Mathematical Sciences, Kyoto University)

Abstract: We consider a random walk in random environment with random holding times, that is, the random walk jumping to one of its nearest neighbors with some transition probability after a random holding time. Both the transition probabilities and the laws of the holding times are randomly distributed over the integer lattice. Our main result is a quenched large deviation principle for the position of the random walk.

The same problem has been studied by Dembo, Gantert, and Zeitouni in one-dimensional case. They assumed that the transition probabilities are uniform elliptic and holding times bounded away from zero but otherwise only quite general ergodicity and integrability conditions. We consider the multidimensional case with rather restrictive independence assumptions: the transition probability and holding times are i.i.d. and mutually independent. On the other hand, we need a weaker ellipticity assumption and also do not assume that holding times are bounded from below. We also assume the so-called "nestling condition" for the transition probability, which allows us to get a simple expression of the rate function.

I will also mention some sub-exponential tail estimates for the slow-down probability.

Based on joint work with Naoki Kubota (Nihon University).

LARGE DEVIATIONS FOR THE LOCAL TIMES OF RANDOM WALK AMONG RANDOM CONDUCTANCES

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We derive an annealed large deviation principle for the normalised local times of a continuous-time random walk among random conductances in a finite state space in the spirit of Donsker-Varadhan. We work in the interesting case that the conductances may assume arbitrarily small values. Thus, the underlying picture of the principle is a joint strategy of small values of the conductances and large holding times of the walk. The speed and the rate function of our principle are explicit in terms of the lower tails of the conductance distribution. As an application, we identify the logarithmic asymptotics of the lower tails of the principal eigenvalue of the randomly perturbed negative Laplace operator in the domain. We discuss also the case where the state space is a subset of the integer lattice and grows with time. Here an interesting phase transition occurs between low and high dimensions.

The talk is based on joint work with Michele Salvi and Tilman Wolff.