DETERMINANTAL MARTINGALES AND CORRELATIONS OF NONCOLLIDING RANDOM WALKS

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We study the noncolliding random walk (RW), which is a particle system of one-dimensional, simple and symmetric RWs starting from distinct even sites and conditioned never to collide with each other. When the number of particles is finite, $N < \infty$, this discrete process is constructed as an h-transform of absorbing RW in the N-dimensional Weyl chamber. We consider Fujita's polynomial martingales of RW with time-dependent coefficients and express them by introducing a complex Markov process. It is a complexification of RW, in which independent increments of its imaginary part are in the hyperbolic secant distribution, and it gives a discrete-time conformal martingale. The h-transform is represented by a determinant of the matrix, whose entries are all polynomial martingales. From this determinantal-martingale representation (DMR) of the process, we prove that the noncolliding RW is determinantal for any initial configuration with $N < \infty$, and determine the correlation kernel as a function of initial configuration. We show that noncolliding RWs started at infinite-particle configurations having equidistant spacing are well-defined as determinantal processes and give DMRs for them. Tracing the relaxation phenomena shown by these infinite-particle systems, we obtain a family of equilibrium processes parameterized by particle density, which are determinantal with the discrete analogues of the extended sine-kernel of Dyson's Brownian motion model with $\beta = 2$. Following Donsker's invariance principle, convergence of noncolliding RWs to the Dyson model is also discussed.

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