GEOMETRIC AND DYNAMICAL RIGIDITY OF STOCHASTIC COULOMB SYSTEMS IN INFINITE-DIMENSIONS

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Stochastic Coulomb dynamics in infinite-dimensions are infinitely many Brownian particles in $\mathbb{R}^d$ interacting via γ-dimensional Coulomb potential Ψ_γ with inverse temperature β. When the systems are translation invariant, then the dynamics are given by the infinite-dimensional stochastic differential equations

$$dX_t^i = dB_t^i - \frac{\beta}{2} \lim_{r\to\infty} \sum_{j\neq i,\, |X_t^i - X_t^j|<r} \nabla\Psi_\gamma(X_t^i - X_t^j)dt \qquad (1)$$

We suppose $d \le \gamma \le d+2$ because Ψ_γ become Ruelle class potentials if $\gamma > d+2$, and it seems difficult to justify the SDEs (1) for $\gamma < d$. If $d \le \gamma \le d+2$, then we call (1) (translation invariant) stochastic Coulomb dynamics. When $\gamma = d$, we call (1) *strict* stochastic Coulomb dynamics. So far the only example of strict stochastic Coulomb dynamics is the Ginibre interacting Brownian motions, namely the case $(\beta, \gamma, d) = (2, 2, 2)$. Dysons models and Airy Interacting Brownian motions are examples of stochastic Coulomb dynamics in one dimension with two-dimensional Coulomb potentials. Namely, $(\beta, \gamma, d) = (1, \gamma, d), (2, \gamma, d), (4, \gamma, d)$. (see [2], [3], [4], [5], [9], [10]).

In this talk, I present various dynamical rigidity of the Ginibre interacting Brownian motions. In particular, I prove that the tagged particles of Ginibre interacting Brownian motions are sub-diffusive [8].

Such a sub-diffusivity of tagged particles may be surprising. For the tagged particles of interacting Brownian motions in $\mathbb{R}^d$ with $d \ge 2$ with Ruelle's class potentials are always diffusive. This diffusivity is proved in [1] for Ruelle's class potentials with convex hard cores. In case of no hard core, this has been not yet fully proved, but generally believed by the specialists in mathematics, and papers in physics are treated as a fact.

In addition, I give a phase transition conjecture on the rigidity of strict stochastic Coulomb dynamics with the inverse temperature β for general dimensions $d \ge 2$.

The proof is based on the results of stochastic geometry developed in [6] and [7].

References

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