Growth exponent for loop-erased random walk in three dimensions

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Abstract

Let S be the simple random walk on $\mathbb{Z}^3$ started at the origin and σ_n be the first time S exits the ball of radius n with center the origin. Let M_n be the number of steps of $\mathrm{LE}(S[0,\sigma_n])$, the loop-erasure of $S[0,\sigma_n]$. Physicists conjecture that there exists α (called the growth exponent) such that $E(M_n) \approx n^{\alpha}$ and did numerical experiments to show that $\alpha = 1.62 \pm 0.01$ ([1]). However, rigorously the existence of α is not proved.

In the talk, I show the existence of the growth exponent, i.e., I prove that there exists α such that

$$\lim_{n\to\infty} \frac{\log E(M_n)}{\log n} = \alpha.$$

References

[1] A. J. Guttmann and R. J. Bursill, Critical exponents for the loop erased self-avoiding walk by Monte Carlo methods, Journal of Statistical Physics 59:1/2 (1990), 1-9.