Strong Markov property of determinatal processes associated with extended kernels

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We denote by $\mathfrak{M}$ the space of nonnegative integer-valued Radon measures on $\mathbb{R}$, which is a Polish space with the vague topology. Any element ξ of $\mathfrak{M}$ can be represented as $\xi(\cdot) = \sum_{j\in\Lambda} \delta_{x_j}(\cdot)$ with a sequence of points in $\mathbb{R}$, $\boldsymbol{x} = (x_j)_{j\in\Lambda}$ satisfying $\xi(K) = \sharp\{x_j : x_j \in K\} < \infty$ for any compact subset $K \subset \mathbb{R}$. The index set $\Lambda = \mathbb{N} \equiv \{1, 2, \dots\}$ or a finite set. We call an element ξ of $\mathfrak{M}$ an unlabeled configuration, and a sequence $\boldsymbol{x}$ a labeled configuration. As a generalization of a notion of determinantal point process on $\mathbb{R}$ for a probability measure on $\mathfrak{M}$, we give the following definition for $\mathfrak{M}$-valued processes.

Definition 1 *An $\mathfrak{M}$-valued process $(\mathbb{P}, \Xi(t), t \in [0,\infty))$ is said to be determinantal with the correlation kernel $\mathbb{K}$, if for any $M \geq 1$, any sequence $(N_m)_{m=1}^M$ of positive integers, any time sequence $0 < t_1 < \cdots < t_M < \infty$, the $(N_1, \dots, N_M)$-multitime correlation function is given by a determinant,*

$$\rho\Big(t_1, \xi^{(1)}; \dots; t_M, \xi^{(M)}\Big) = \det_{\substack{1\leq j\leq N_m, 1\leq k\leq N_n \\ 1\leq m,n\leq M}} \left[\mathbb{K}(t_m, x_j^{(m)}; t_n, x_k^{(n)})\right],$$

where $\xi^{(m)}(\cdot) = \sum_{j=1}^{N_m} \delta_{x_j^{(m)}}(\cdot), 1 \leq m \leq M$.

We consider the determinatal processes $(\mathbb{P}^\xi, \Xi(t))$ associated with the extended sine kernel, extended Airy kernel and extended Bessel kernel. These processes are reversible Markov process [1]. In this talk we discuss the following:

1. The Strong Markov property of the processes
2. The SDEs and Dirichlet forms related to the processes

References

[1] Katori, M., Tanemura, H.: Markov property of determinantal processes with extended sine, Airy, and Bessel kernels. Markov process Relat. Fields **17**, 541-580 (2011)

[2] Osada, H., Tanemura, H.: Strong solutions of infinite-dimensional stochastic differential equations and tail σ-fields. (in preparation)