Eigenvalue fluctuations for lattice Anderson Hamiltonians

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Consider a lattice Anderson Hamiltonian

$$H_{D_\epsilon,\xi} := -\epsilon^{-2}\Delta^{(\mathrm{d})} + \xi^{(\epsilon)}, \tag{0.1}$$

where $\Delta^{(\mathrm{d})}$ is the standard lattice Laplacian and $\xi^{(\epsilon)}$ is a random field on $\mathbb{Z}^d$ acting as multiplication operator. We impose to this operator the Dirichlet boundary condition outside

$$D_\epsilon := \left\{x \in \mathbb{Z}^d\colon\ \mathrm{dist}_\infty(\epsilon x, D^{\mathrm{c}}) > \epsilon\right\} \tag{0.2}$$

with a smooth bounded open subset $D \subset \mathbb{R}^d$ and study the asymptotics of the eigenvalues $\{\lambda^{(k)}_{D_\epsilon,\xi}\}_{k\geq 1}$ ordered increasingly. Our assumptions on the random potential $\xi^{(\epsilon)}$ are as follows:

Assumption 1. (1) *$(\{\xi^{(\epsilon)}(x)\}_{x\in D_\epsilon}, \mathbb{P})$ is independent and $\mathbb{E}\left(|\xi^{(\epsilon)}(x)|^K\right)$ is bounded in $x \in D_\epsilon$ and $\epsilon \in (0,1)$ for some $K > 2 \vee d/2$;*

(2) *There are bounded continuous functions U and V on D such that for each $\epsilon > 0$ and $x \in D_\epsilon$, $\mathbb{E}\xi^{(\epsilon)}(x) = U(\epsilon x)$ and $\mathrm{Var}\big(\xi^{(\epsilon)}(x)\big) = V(\epsilon x)$.*

Let us denote by $\lambda^{(k)}_D$ and $\varphi^{(k)}_D$ the k-th smallest eigenvalue/eigenfunction of the operator $-\Delta + U$ with the Dirichlet boundary condition outside D. The first result is the convergence of the random eigenvalues to the continuum limit.

Theorem 1. *Under Assumption 1, for each $k \geq 1$, $\lambda^{(k)}_{D_\epsilon,\xi} \to \lambda^{(k)}_D$ as $\epsilon \downarrow 0$ in probability.*

The second result is a Gaussian asymptotic limit law for the fluctuation of $\lambda^{(k)}_{D_\epsilon,\xi}$ around its mean. We will do this jointly for the collection of all simple eigenvalues:

Theorem 2. *Suppose Assumption 1 holds and $k_1, \dots, k_n \in \mathbb{N}$ are distinct indices such that the eigenvalues $\lambda^{(k_1)}_D, \dots, \lambda^{(k_n)}_D$ of $H_{D,U}$ are simple. Then, in the limit as $\epsilon \downarrow 0$, the random vector*

$$\left(\frac{\lambda^{(k_1)}_{D_\epsilon,\xi} - \mathbb{E}\lambda^{(k_1)}_{D_\epsilon,\xi}}{\epsilon^{d/2}}, \dots, \frac{\lambda^{(k_n)}_{D_\epsilon,\xi} - \mathbb{E}\lambda^{(k_n)}_{D_\epsilon,\xi}}{\epsilon^{d/2}}\right) \tag{0.3}$$

converges in distribution to a multivariate normal with mean zero and covariance

$$\sigma^2_{ij} := \int_D \varphi^{(k_i)}_D(x)^2 \varphi^{(k_j)}_D(x)^2\, V(x)\,\mathrm{d}x. \tag{0.4}$$