Dynamical convergence of infinite particle system related to random matrices

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Consider the infinite particle system in $\mathbb{R}$ interacting through log potential described by the following infinite-dimensional stochastic differential equation (ISDE)

$$dX_t^i = dB_t^i + \lim_{r\to\infty} \sum_{|X_t^i - X_t^j|<r} \frac{1}{X_t^i - X_t^j} dt \quad (i \in \mathbb{N}), \tag{1}$$

where B_t^i is Brownian motion. Osada proved that there exists a solution of this ISDE.

In this talk, we consider a natural finite particles approximation of the ISDE. Let μ_θ^N be a distribution of unlabeled particle system θ-shifted Gaussian Unitary Ensemble at the bulk scaling. The labeled density of μ_θ^N is by definition

$$\mu_\theta^N(d\mathbf{x}_N) = \frac{1}{Z} \prod_{i<j}^{N} | x_i - x_j |^2 \prod_{k=1}^{N} e^{-\frac{|x_k - N\theta|^2}{4N}} d\mathbf{x}_N.$$

It is known that μ_θ^N has a universality in the sense that μ_θ^N converge to the sine random point field $\mu_{sin,2}$ in distribution for $-\sqrt{2} < \theta < \sqrt{2}$. Here $\mu_{sin,2}$ is determinantal random point field with kernel

$$K(x,y) = \frac{\sin\{\sqrt{2-\theta^2}(x-y)\}}{\pi(x-y)}.$$

We consider the dynamical counterpart of this fact. The SDE associated with μ_θ^N is given by

$$dX_t^{N,i} = dB_t^{N,i} + \sum_{1\le j\ne i\le N} \frac{1}{X_t^{N,i} - X_t^{N,j}} dt - \frac{1}{2N} X_t^{N,i} dt + \frac{\theta}{2} dt \quad (1 \le i \le N). \tag{2}$$

Let $l^N = (l_1^N, ..., l_N^N)$ and $l = (l_1, l_2, ...)$ be labels, that is, a functions from a configuration space $\mathsf{S} = \{\mathsf{s} = \sum \delta_{s_i}; s_i \in \mathbb{R}, \mathsf{s} \text{ is a Radom measure}\}$ to $\cup_n \mathbb{R}^n \cup \mathbb{R}^{\mathbb{N}}$. Define $l_m^N = (l_1^N, ..., l_m^N)$ and $l = (l_1, l_2, ...l_m)$ as the first m particle of the labels l^N, l respectively. Let $(X^{N,1}, X^{N,2}, ..., X^{N,N})$ be a solution of (2) starting at $l^N(\mathsf{s})$ and $(X^1, X^2, ...)$ be a solution of (1) starting at $l(\mathsf{s})$.

Theorem 0.1. *Suppose* $\lim_{N\to\infty} \mu_\theta^N \circ (l_m^N)^{-1} = \mu_{sin,2} \circ (l_m)^{-1}$ *weakly. Then for each m, the convergence of the first m particle to that of ISDE* (1)

$$\lim_{N\to\infty} (X^{N,1}, X^{N,2}, ..., X^{N,m}) = (X^1, X^2, ..., X^m) \text{ weakly in } C([0,\infty), \mathbb{R}^m).$$

Here, the limit ISDE (1) is independent of θ. This phenomenon is due to the geometric universality of random matrix explained above.

This is joint work with H. Osada.