An estimate of spectral gap for surface diffusion.

Yukio Nagahata
Department of Information Engineering Faculty of Engineering, Niigata University, JAPAN
nagahata@ie.niigata-u.ac.jp

Surface diffusion is one of the evolutional model of discrete surface. The surface is associated with two dimensional Young diagram. There is a bijection between two dimensional Young diagram and the configuration of the exclusion process. Hence we formulate this process as an exclusion process.

We set $\Lambda_n = \{1, 2, \dots n\}$. We define the set of configurations by $\Sigma_n := \{0,1\}^{T_n}$, the set of configurations conditioned by the number of particles and moment by $\Sigma_{n,K,M} := \{\eta \in \Sigma_n; \sum_{x\in\Lambda_n} \eta_x = K, \sum_{x\in\Lambda_n} x\eta_x = m\}$.

For $\eta \in \Sigma_n$ and $i, j \in \Lambda_n$, $i < j$, we define the configuration $\sigma^{(i,j)}\eta \in \Sigma_n$ by $(\sigma^{(i,j)}\eta)_{i-1} = \eta_i$, $(\sigma^{(i,j)}\eta)_i = \eta_{i-1}$, $(\sigma^{(i,j)}\eta)_j = \eta_{j+1}$, $(\sigma^{(i,j)}\eta)_{j+1} = \eta_j$, and $(\sigma^{(i,j)}\eta)_k = \eta_k$, for $k \neq i-1, i, j, j+1$, and the operator $\pi^{(i,j)}$ by $\pi^{(i,j)} f(\eta) = f(\sigma^{(i,j)}) - f(\eta)$.

We set $F_r^{\pm}, G^{\pm}$ as

$$
\begin{aligned}
F_r^+(\eta) &:= \mathbf{1}(\eta_{-1} = 0, \eta_0 = \ldots = \eta_r = 1, \eta_{r+1} = 0)(\eta) \\
F_r^-(\eta) &:= \mathbf{1}(\eta_{-1} = 1, \eta_0 = 0, \eta_1 = \ldots = \eta_{r-1} = 1, \eta_r = 0, \eta_{r+1} = 1)(\eta) \\
G_r^-(\eta) &:= \mathbf{1}(\eta_{-1} = 1, \eta_0 = \ldots = \eta_r = 0, \eta_{r+1} = 1)(\eta) \\
G_r^+(\eta) &:= \mathbf{1}(\eta_{-1} = 0, \eta_0 = 1, \eta_1 = \ldots = \eta_{r-1} = 0, \eta_r = 1, \eta_{r+1} = 0)(\eta),
\end{aligned}
$$

and define $c(x, y; \eta)$ by

$$
c(x, y; \eta) := \frac{1}{|y-x|^2}\{\tau_x(F_{y-x}^+(\eta) + F_{y-x}^-(\eta)) + \tau_x(G_{y-x}^+(\eta) + G_{y-x}^-(\eta))\},
$$

here τ_x is a shift operator. Our generator is defined by

$$
Lf(\eta) := \sum_{x,y;x<y} c(x, y; \eta)\pi^{(x,y)} f(\eta).
$$

It is easy to see that uniform measure on $\Sigma_{n,K,M}$ is a reversible measure. Let $L_{n,K,M}$ be the restriction of L on $\Sigma_{n,K,M}$. Then we can consider the spectral gap of $-L_{n,K,M}$, which is defined by

$$
\lambda = \lambda(n, K, M) := \inf\left\{ \frac{E[f(-L_{n,K,M})f]}{E[f^2]} \middle| E[f] = 0 \right\}.
$$

We shall state conjecture and our result.

Conjecture 1 There exists a constant C not depending on n, K nor M such that

$$
\lambda(n, K, M) \geq \frac{C}{n^4}
$$

Proposition 2 There exists a constant C not depending on n, K nor M such that

$$
\lambda(n, K, M) \geq \frac{C}{n^5}
$$