

# The $q$ -Hahn zero range process and TASEP

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For the one-dimensional KPZ equation, a compact formula was found for a certain generating function of the height function. In one approach known as the replica method, the formula is obtained using a connection to the problem of  $\delta$ -Bose gas. In the derivation of the formula, one needs to use the completeness of the eigenfunctions of the system. This is not at all trivial but has been established for the  $\delta$ -Bose gas..

The  $q$ -Hahn totally asymmetric zero range process (TAZRP) is a version of discrete time zero range process in which the particle hopping probability depends on a function,

$$\varphi_{q,\mu,\nu}(j|m) = \mu^j \frac{(\nu/\mu; q)_j (\mu; q)_{m^j} (q; q)_m}{(\nu; q)_j (q; q)_j (q; q)_{m-j}}.$$

This is related to a measure for which the  $q$ -Hahn polynomials are orthogonal. Hence we call it the  $q$ -Hahn TAZRP. By a standard mapping between the zero range process and exclusion process, one can also define the corresponding totally asymmetric simple exclusion process (TASEP). In this process, a particle hops in one direction with the hopping probability given by the same function under the exclusion rule.

As a special limit, the  $q$ -Hahn TAZRP reduces to the  $q$ -boson TAZRP, which had been studied earlier (at the same time the  $q$ -Hahn TASEP reduces to the  $q$ -TASEP). In a further limit, they reduce to the KPZ equation. Hence the  $q$ -Hahn processes can be considered as a discretized version of the KPZ equation.

By using the Bethe ansatz, we construct the eigenfunctions of the generator explicitly and establish the Plancherel theorem. From this follow immediately the completeness and the orthogonality relations of the eigenfunctions. By using the properties, we show for a certain class of initial conditions the  $q$ -moment for the height function can be written as multiple integrals with nested contours.

The presentation is based on a collaboration with A. Borodin, I. Corwin, L. Petrov.

[BCPS] A. Borodin, I. Corwin, L. Petrov, T. Sasamoto, Spectral theory for interacting particle systems solvable by coordinate Bethe ansatz, arXiv:1407.8534.