An invariance principle for stochastic heat equations with periodic coefficients

Lu Xu (The University of Tokyo)

In this talk we extend the central limit theorem for finite dimensional diffusion processes to infinite dimensional settings. Consider a stochastic heat equation on $[0,1]$ with a homogeneous Neumann's boundary condition and driven by a space-time white noise, written as:

$$\begin{cases} \partial_t u(t,x) = \frac{1}{2}\partial_x^2 u(t,x) - V_x'(u(t,x)) + \dot{W}(t,x), & t>0, x\in(0,1), \\ \partial_x u(t,0) = \partial_x u(t,1) = 0, & t>0, \\ u(0,x) = v(x) \in C([0,1]), & x\in[0,1]. \end{cases}$$

Obviously the mild solution $u(t,\cdot)$ forms a $C([0,1])$-valued, continuous Markov process whose invariant measure can be given by

$$\mu(dv) = \exp\left\{-2\int_0^1 V_x(v(x))dx\right\}\mu_w[d(v-v(0))]\otimes d[v(0)],$$

where μ_w is the standard Wiener measure on $C_0([0,1])$ and d is the Lebesgue measure on $\mathbb{R}$.

We assume that $\{V_x(\cdot), x\in[0,1]\}$ is a group of periodic functions belonging to $C_b^2(\mathbb{R})$ such that $V_x(u) = V_x(u+1)$, and investigate the asymptotic behaviors of $u(t,x)$. We prove that for every $T>0$ and any initial probability measure $\nu \ll \mu$,

$$\left\{\epsilon u(\epsilon^{-2}t,\cdot)\right\}_{t\in[0,T]} \xrightarrow{\epsilon\downarrow 0} \{\sigma B_t \cdot \mathbf{1}\}_{t\in[0,T]}$$

in the topology of $C([0,T], C([0,1]))$, where σ is a constant only depending on V_x, B_t is a 1-d standard Brownian motion and $\mathbf{1}$ stands for the constant function on [0, 1] defined by $\mathbf{1}(x)\equiv 1$. We also compute out the diffusion constant σ.

The proof is based on the general theory of central limit theorem for additional functionals of reversible and ergodic Markov process developed by C. Kipnis and S. Varadhan, and an extended Itô's formula related to our equation.