Non-equilibrium macroscopic evolution of chain of oscillators with conservative noise

Stefano Olla

In the non-equilibrium evolution of a one dimensional chain of anharmonic oscillators we expect two main space-time scales: a hyperbolic scale where the evolution is ballistic-mechanic, dominated by tension gradients and governed by Euler equations, and a super-diffusive scale where the evolution, at constant tension, depends on the gradients of the temperature and is governed by a fractional heat equation. This conjecture can be proven for a harmonic chain with random exchange of momentum between nearest neighbor particles. Non-acoustic chains (tensionless) will instead behave diffusively.

Dualities for asymmetric interacting particle systems

Tomohiro Sasamoto

(Self-)Duality has been known to be a useful tool for studying stochastic interacting particle systems. A well-known example is the self-duality for symmetric simple exclusion process. It implies that the n-point correlation functions satisfy the n-particle evolution equation of the same process and is known to be related to the SU(2) symmetry of the process. Another example with a self-duality is the Kipnis-Marchioro-Pressutt(KMP) model with SU(1,1) symmetry.

There had been much less studies on (self-)dualities for asymmetric processes, but recently it turned out that the self-duality for the one-dimensional asymmetric simple exclusion process (ASEP), which is related to the q-deformed symmetry $U_q(sl_2)$, provides a very effective way to study fluctuations of the current and other quantities.

Recently we have presented a rather general scheme to construct asymmetric interacting processes with a given deformed symmetry[CGRS]. We will explain this and give a few examples.

A basic idea is that the (self-)duality is related to the existence of a symmetry, i.e., an operator which commutes with the generator. By starting from a deformed symmetry, one may first construct a real symmetric operator with some symmetry. Then applying a ground state transformation, one gets a generator of an asymmetric process with a transformed symmetry.

The frst example is a generalization of ASEP, in which there could be more than one particles. This is related to the spin-j representation of the $U_q(sl_2)$ with the special case $j = 1/2$ corresponding to the usual ASEP.

As a second example we take $U_q(su(1,1))$ and construct new asymmetric processes. In a certain limit, the model tends to an asymmetric version of the KMP model, whose generator is given in the following form,

$$
\begin{aligned}
&L^{AKMP(\sigma)} f(x) \\
&= \sum_{i=1}^{L-1} \Big\{ \frac{2\sigma(x_i + x_{i+1})}{e^{2\sigma(x_i+x_{i+1})} - 1} \quad \int_0^1 [f(x_1, \dots, w(x_i + x_{i+1}), (1-w)(x_i + x_{i+1}), \dots, x_L) - f(x)] \\
&\times e^{2\sigma w(x_i+x_{i+1})}\, dw \Big\}
\end{aligned}
$$

The presentation is based on a collaboration with G. Carinci, C. Giardina, F. Redig.

[CGRS] G. Carinci, C. Giardina, F. Redig and T. Sasamoto, A generalized Asymmetric Exclusion Process with $U_q(sl_2)$ stochastic duality, arXiv: 1407.3367; Asymmetric stochastic transport models with $U_q(su(1,1))$ symmetry, 1507.01478.

A new technique for the computation of central limit theorem variances for exclusion processes and its application

Makiko Sasada

To show the hydrodynamic limit or equilibrium fluctuation for a non-gradient system, it is believed that we need to show a sharp spectral gap estimate to apply the Varadhan's non-gradient method. The estimate is used to characterize the closed forms. In this talk, we give a new proof for the characterization of closed forms without the spectral gap estimate for exclusion processes reversible under Bernoulli measures, which is the model studied by Funaki, Uchiyama and Yau (1996). We also show an application of this result to the characterization of closed forms for exclusion processes in crystal lattices.

Macroscopic diffusion in random Lorentz gases

Raphael Lefevere

We consider a the mirrors model in a finite d-dimensional domain and connected to particles reservoirs at fixed chemical potentials. The dynamics is purely deterministic and non-ergodic. We study the macroscopic current of particles in the stationary regime. We show first that when the size of the system goes to infinity, the behaviour of the stationary current of particles is governed by the proportion of orbits crossing the system. Using this approach, it is possible to give a rigorous proof Fick's law in a simplified version of the mirrors model in high-dimension. In the mirrors model itself, numerical simulations indicate the validity of Fick's law in three dimensions and above.

An estimate of spectral gap for surface diffusion.

Yukio Nagahata, nagahata@ie.niigata-u.ac.jp

Surface diffusion is one of the evolutional model of discrete surface. The surface is associated with two dimensional Young diagram. There is a bijection between two dimensional Young diagram and the configuration of the exclusion process. Hence we formulate this process as an exclusion process.

We set $\Lambda_n = \{1, 2, \dots n\}$. We define the set of configurations by $\Sigma_n := \{0,1\}^{\Lambda_n}$, the set of configurations conditioned by the number of particles and moment by $\Sigma_{n,K,M} := \{\eta \in \Sigma_n; \sum_{x\in\Lambda_n} \eta_x = K, \sum_{x\in\Lambda_n} x\eta_x = M\}$.

For $\eta \in \Sigma_n$ and $i, j \in \Lambda_n$, $i < j$, we define the configuration $\sigma^{(i,j)}\eta \in \Sigma_n$ by $(\sigma^{(i,j)}\eta)_{i-1} = \eta_i$, $(\sigma^{(i,j)}\eta)_i = \eta_{i-1}$, $(\sigma^{(i,j)}\eta)_j = \eta_{j+1}$, $(\sigma^{(i,j)}\eta)_{j+1} = \eta_j$, and $(\sigma^{(i,j)}\eta)_k = \eta_k$, for $k \neq i-1, i, j, j+1$, and the operator $\pi^{(i,j)}$ by $\pi^{(i,j)} f(\eta) = f(\sigma^{(i,j)}\eta) - f(\eta)$.

We set $F_r^{\pm}, G^{\pm}$ as

$$\begin{aligned}
F_r^+(\eta) &:= \mathbf{1}(\eta_{-1} = 0, \eta_0 = \dots = \eta_r = 1, \eta_{r+1} = 0)(\eta) \\
F_r^-(\eta) &:= \mathbf{1}(\eta_{-1} = 1, \eta_0 = 0, \eta_1 = \dots = \eta_{r-1} = 1, \eta_r = 0, \eta_{r+1} = 1)(\eta) \\
G_r^-(\eta) &:= \mathbf{1}(\eta_{-1} = 1, \eta_0 = \dots = \eta_r = 0, \eta_{r+1} = 1)(\eta) \\
G_r^+(\eta) &:= \mathbf{1}(\eta_{-1} = 0, \eta_0 = 1, \eta_1 = \dots = \eta_{r-1} = 0, \eta_r = 1, \eta_{r+1} = 0)(\eta),
\end{aligned}$$

and define $c(x, y; \eta)$ by

$$c(x, y; \eta) := \frac{1}{|y-x|^2}\{\tau_x(F_{y-x}^+(\eta) + F_{y-x}^-(\eta)) + \tau_x(G_{y-x}^+(\eta) + G_{y-x}^-(\eta))\},$$

here τ_x is a shift operator. Our generator is defined by

$$Lf(\eta) := \sum_{x,y;x<y} c(x,y;\eta)\pi^{(x,y)} f(\eta).$$

It is easy to see that uniform measure on $\Sigma_{n,K,M}$ is a reversible measure. Let $L_{n,K,M}$ be the restriction of L on $\Sigma_{n,K,M}$. Then we can consider the spectral gap of $-L_{n,K,M}$, which is defined by

$$\lambda = \lambda(n,K,M) := \inf\left\{ \frac{E[f(-L_{n,K,M})f]}{E[f^2]} \middle| E[f] = 0 \right\}.$$

We shall state our result.

Theorem 1. There exists a constant C not depending on n, K nor M such that

$$\lambda(n,K,M) \geq \frac{C}{n^4}$$

On phase transition of random walk pinning model

Makoto Nakashima

In this talk, we consider the discrete random walk pinning model (RWPM), which was introduced by Birkner and Sun [3].

Let X and Y be two independent simple random walks on $\mathbb{Z}^d$. We denote by P_X^x and P_Y^y the law of X and Y starting from x and y. Especially, we denote by $P_X = P_X^0$ and $P_Y = P_Y^0$. Then, RWPM is defined as follows:

Definition 1. *Fix fixed Y and $\beta \geq 0$. Then, we define a new path measure $\mu_{N,Y}^\beta$ by*

$$\mu_{N,Y}^\beta(dX) = \frac{1}{Z_{N,Y}^\beta} \exp\left(\beta L_N(X,Y)\right) P_X^0(dX),$$

where

$$Z_{N,Y}^\beta = P_X\left[\exp(\beta L_N(X,Y))\right]$$

is called a quenched partition function and

$$L_N(X,Y) = \sum_{k=1}^N \mathbf{1}\{X_k = Y_k\}$$

is a collision local time between X and Y up to time N.

Remark: We can regard X as a polymer shape and Y as a fluctuating interface or moving catalytists.

Also, we introduce the annealed partition function as the P_Y-mean of $Z^{\beta}_{N,Y}$.

The monotone convergence theorem implies the existence of the limits of the quenched and the annealed partition functions:

$$Z^{\beta}_{Y} := \lim_{N\to\infty} Z^{\beta}_{N,Y} \qquad\qquad P_Y[Z^{\beta}_{Y}] := \lim_{N\to\infty} P_Y[Z^{\beta}_{N,Y}].$$

Moreover, the monotonicity of the limits in β yields the phase transitions:

$$\beta^q_1(d) = \sup\{\beta \geq 0 : Z^{\beta}_{Y} < \infty, \quad P_Y\text{-a.s.}\} \qquad\qquad \beta^a_1(d) = \sup\{\beta \geq 0 : P_Y[Z^{\beta}_{Y}] < \infty\}.$$

Then, it is clear that $\beta^a_1(d) \leq \beta^q_1(d)$.

Also, we can consider another phase transition described in terms of the free energies which are important quantities given by

$$\begin{aligned} F^q(\beta) &= \lim_{N\to\infty} \frac{1}{N} \log Z^{\beta}_{N,Y} = \lim_{N\to\infty} \frac{1}{N} P_Y[\log Z^{\beta}_{N,Y}] \\ F^a(\beta) &= \lim_{N\to\infty} \frac{1}{N} \log P_Y[Z^{\beta}_{N,Y}]. \end{aligned}$$

Then, the monotonicity of the free energies in β yields the phase transitions:

$$\beta^q_2(d) = \sup\{\beta \geq 0 : F^q(\beta) = 0\} \qquad\qquad \beta^a_2(d) = \sup\{\beta \geq 0 : F^a(\beta) = 0\}.$$

It is known that

$$\begin{aligned} &\beta^a_1(d) = \beta^a_2(d) = \beta^q_1(d) = \beta^q_2(d) = 0 && \text{if } d = 1, 2 \\ &0 < \beta^a_1(d) = \beta^a_2(d) < \beta^q_1(d) \leq \beta^q_2(d) && \text{if } d \geq 3 \end{aligned}$$

[1, 2, 3, 4]

Theorem. $\beta^q_1(d) = \beta^q_2(d)$ *for any* $d \geq 3$.

Thus, we know the coincidence of the critical points. Moreover, we have some variational representation of the quenched and annealed free energies. The variational representation gives us a some asymptotics of free energy at high temperature in the case $d = 1, 2$.

Theorem. *When* $\beta \searrow 0$,

$$\begin{aligned} F^q(\beta) &\asymp \beta^2, \quad d = 1 \\ \log F^q(\beta) &\asymp -\beta^{-2}, \quad d = 2 \end{aligned}$$

In the last, we remark that the behavior of X under $\mu^{\beta}_{N,Y}$ is definitely different between β beyond and above the critical point. Especially, we know that X satisfies the central limit theorem when β is beyond the critical point.

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Recurrence and transience of multi-dimensional Brox-type diffusion processes

Hiroshi Takahashi

We consider limiting behavior of multi-dimensional diffusion processes in stable and semi-stable Lévy environments. Let $\mathcal{W}$ be the space of the $\mathbf{R}$-valued functions W that satisfy (I) $W(0) = 0$, (II) W is right continuous and has left limits on $[0, \infty)$, and (III) W is left continuous and has right limits on $(-\infty, 0]$. We set a probability measure Q on $\mathcal{W}$ such that $\{W(x), x \geq 0, Q\}$ and $\{W(-x), x \geq 0, Q\}$ are independent strictly semi-stable Lévy processes with index $\alpha \in (0, 2]$, which have the following semi-selfsimilarity:

$$\{W(x), x \in \mathbf{R}\} \overset{d}{=} \{a^{-1/\alpha} W(ax), x \in \mathbf{R}\} \quad \text{for some } a > 0, \tag{0.1}$$

where $\overset{d}{=}$ denotes the equality in all joint distributions.

For a fixed W, we consider a d-dimensional diffusion process starting at 0, $X_W = \{X_W^k(t), t \geq 0, k = 1, 2, 3, \ldots, d\}$ whose generator is

$$\sum_{k=1}^{d} \frac{1}{2} \exp\{W(x_k)\} \frac{\partial}{\partial x_k} \left\{ \exp\{-W(x_k)\} \frac{\partial}{\partial x_k} \right\}. \tag{0.2}$$

Each component of X_W is symbolically described by

$$dX_W^k(t) = dB^k(t) - \frac{1}{2}W'(X_W^k(t))dt, \quad X_W^k(0) = 0, \quad \text{for } k = 1, 2, 3, \dots, d,$$

where $B^k(t)$ is a one-dimensional standard Brownian motion independent of the environment (W, Q). In the case where $d = 1$ and $\{W(x)\}$ is a Brownian motion, this diffusion process is studied by Brox [1]. He showed that the distribution of $(\log t)^{-2} X_W(t)$ converges weakly as $t \to \infty$. Our main theorem is as follows:

Theorem. (i) *If $\{-W(x), Q\}$ is not a subordinator, then X_W is recurrent for almost all environments in any dimension.*

(ii) *If $\{-W(x), Q\}$ is a subordinator, then X_W is transient for almost all environments in any dimension.*

This is a joint work with KUSUOKA, Seiichiro (Tohoku University) and TAMURA, Yozo (Keio University).

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Static and dynamic large deviations for a reaction-diffusion model

Kenkichi Tsunoda

We consider in this talk interacting particle systems in which a symmetric simple exclusion dynamics, speeded-up diffusively, is superposed to a non-conservative Glauber dynamics. De Masi, Ferrari and Lebowitz [1] proved that the macroscopic evolution of the empirical measure is described by the solutions of the reaction-diffusion equation

$$\partial_t \rho \;=\; (1/2)\Delta\rho + B(\rho) - D(\rho)\;.$$

where Δ is the Laplacian and $F = B - D$ is a reaction term determined by the stochastic dynamics. The main results of our study concern the large deviations of the Glauber+Kawasaki dynamics. We present the static and dynamic large deviations principle for the empirical measure under the assumption that B and D are concave functions. These assumptions encompass the case in which the potential $F(\rho) = B(\rho) - D(\rho)$ presents two or more wells, and open the way to the investigation of the metastable behavior of this dynamics. We also present the properties that the large deviations rate function is lower semicontinuous and has compact level sets. These properties play a fundamental role in the proof of the static large deviation principle for the empirical measure under the stationary state. This talk is based on joint works with Jonathan Farfan and Claudio Landim [3, 2].

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Central limit theorem for stochastic heat equations in random environments

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In this talk, we consider a 1-dimensional stochastic heat equation living in a random environment which is generated by a stationary, ergodic random field. We prove through the method of environmental process, that as time tends to infinity, the solution satisfies the central limit theorem in probability with respect to the environment. The limit distribution is a centered Gaussian law concentrating only on constant functions.

Precisely, let $H = L^2[0,1]$ and consider a $\mathbb{R} \times H$-valued random field

$$\{(V(\sigma, u), B(\sigma, u)) \in \mathbb{R} \times H; \sigma \in \Sigma, u \in H\}$$

on some probability space $(\Sigma, \mathscr{A}, Q)$. Let $\dot{W}(t,x)$ be a standard space-time white noise constructed on another probability space $(\Omega, \mathscr{F}, P)$. For a fixed $\sigma \in \Sigma$, consider a 1-d stochastic heat equation, depending on σ, with homogeneous Neumann's boundary condition:

$$\begin{cases} \partial_t u^\sigma(t,x) = \frac{1}{2}\partial_x^2 u^\sigma(t,x) - DV(\sigma, u^\sigma(t)) - B(\sigma, u^\sigma(t)) + \dot{W}(dt,dx), & t>0, x\in(0,1); \\ \partial_x u^\sigma(t,x)|_{x=0} = \partial_x u^\sigma(t,x)|_{x=1} = 0, & t>0; \\ u^\sigma(0,x) = v(x), & x \in [0,1], \end{cases}$$

where D is the Fréchet differentiable operator on H. Supposing that $DV + B$ is bounded and Lipschitz continuous, we have the solution $u^\sigma(\omega; t, \cdot) \in C[0,1]$ almost surely. Then

$$u(t) : (\omega, \sigma) \mapsto u(\omega, \sigma; t) \triangleq u^\sigma(\omega; t, \cdot)$$

defines a stochastic process on the product space $(\Omega \times \Sigma, \mathscr{F} \otimes \mathscr{A})$. Assume that (V, B) is stationary and ergodic under constant-shifts and B is diffusion free, i.e.,
(i) $\forall f \in C_b^1(H; \mathbb{R})$, $\int_{E_0} e^{-2V(\sigma,u)}\langle Df(u), B(\sigma,u)\rangle \mu_0(du) = 0$, Q-a.s, where E_0 is the space of continuous functions on $[0,1]$ vanishing at 0, and μ_0 is the distribution of a standard 1-d Brownian motion on $[0,1]$ starting from the origin.
(ii) The distribution of (V, B) on the path space is stationary and ergodic under the transfer semigroup $\{\tau_c; c \in \mathbb{R}\}$ defined by

$$\tau_c \phi \triangleq \phi(\cdot + c), \quad \forall \phi : H \to \mathbb{R} \times H.$$

Theorem (CLT in probability w.r.t the environment)**.** *Assume (i) and (ii) , then for every bounded and continuous functional f on $C[0,1]$, we have*

$$\lim_{t\to\infty} E_Q \left| E_P \left[f\left(\frac{u^\sigma(t)}{\sqrt{t}} \right) \right] - \int_{\mathbb{R}} f(\mathbf{1} \cdot y) \Phi_a(y) dy \right| = 0,$$

where $\mathbf{1}$ is the constant function over $[0,1]$ such that $\mathbf{1}(x) \equiv 1$, $a = a(V,B)$ is some positive constant, and Φ_a is the probability density function of 1-d centered Gaussian law with covariance a^2. Furthermore, there exists some constant $C = C(V) > 0$ such that $a^2 \in [C, 1]$.

Invariance Principle for the Random Conductance Model with dynamic degenerate weights

Jean-Dominique Deuschel

In this talk we present a quenched invariance principle for the dynamic random conductance model, that is we consider a continuous time random walk on $\mathbb{Z}^d$ in an environment of time-dependent random conductances. We assume that the conductances are stationary ergodic with respect to space-time shifts and satisfy some moment conditions. One key result in the proof is a maximal inequality for the corrector function, which is obtained by a Moser iteration. This is joint work with Sebastian Andres, Alberto Chiarini,and Martin Slowik.

On an extension of the Brascamp-Lieb inequality

Yuu Hariya

The Brascamp-Lieb moment inequality plays an important role in statistical mechanics. It asserts that the centered moments of a Gaussian distribution perturbed by a convex potential do not exceed those of the Gaussian distribution. In this talk, we give a probabilistic proof of the Brascamp-Lieb inequality based on Skorokhod embedding; error bounds for the inequality in terms of the variance are provided as well. The same reasoning also enables us to extend the inequality to a relatively wide class of nonconvex potentials including double-well potentials in the case of one dimension.

This talk is based on [1] and [2, Appendix].

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Inverse Problems for Stochastic Transport Equations

Yoshiki OTOBE

We shall discuss inverse problems for stochastic transport equations of additive noise case:

$$\begin{aligned} \partial_t u(t,x) &= \partial_x u(t,x) + V(x)u(t,x) + \dot{W}, \\ u(0,x) &= u_0(x), \end{aligned}$$

or linear case:

$$\begin{aligned} \partial_t u(t,x) &= \partial_x u(t,x) + [V(x) + \dot{W}]u(t,x), \\ u(0,x) &= u_0(x), \end{aligned}$$

where $u_0(x)$ is a known initial state, and $V(x)$ is an unknown function describing a medium. Here the noise $\dot{W}$ is assumed to be either temporal or spatial Gaussian white. Our aim is to recover the function $V(x)$ from a single observation data $u(a,t)$ at some $a \in \mathbb{R}$. Note that $V(x)$ must be deterministic while the observation is random. In this talk we shall mainly focus on a method using a quadratic variation process of the observation rather than using the law of large numbers which was announced before in an early stage to recover a deterministic quantity from a random one. The talk is based on a joint work with Dan Crisan (Imperial College London) and Szymon Peszat (Polish Academy of Sciences).

Φ^4 model on large scales

Hendrik WEBER

The theory of non-linear stochastic PDEs has recently witnessed an enormous breakthrough when Hairer and Gubinelli devised methods to give an interpretation and show local well-posedness for a class of very singular SPDEs from mathematical physics.

In this talk I will discuss how to extend their method to get global bounds in a prominent example, the dynamic Φ^4 model. I will first show how to use a simple PDE argument to show global in time well-posedness for Φ^4 equation on the two-dimensional plane. In a second step I will show how to extend this method to get a global in time solutions for the three dimensional Φ^4 model on a torus.

This is joint work with Jean-Christophe Mourrat.

Paracontrolled distributions and the KPZ equation

Massimiliano Gubinelli

In this talk we will review the remarkable solution of the KPZ equation given by Hairer in the language of paracontrolled distribution. In particular we will discuss some related results: Hopf–Cole transformation to the Rough heat equation, the relation with energy solutions of Jara and Gonçalves, convergence of discretize models of the Sasamoto–Spohn type, the related stochastic control problem and Delarue–Diehl diffusion, global existence of solutions and positivity of the Rough multiplicative heat equation. If time permit I will comment also on the weak universality problem for KPZ. Based on joint work with N. Perkowski: M. Gubinelli and N. Perkowski. KPZ reloaded. arXiv:1508.03877.

Extremes of local times for simple random walks on symmetric trees

Yoshihiro Abe

In this talk I will consider local times of the simple random walk on the b-ary tree of height n at times much larger than the maximal hitting time. I will describe results about weak convergence of the maximum of the local times over leaves and a point process of the local extrema as n goes to infinity. The limit objects are characterized by the so-called derivative martingale for the corresponding branching random walk.

Geometric structures of favorite points and late points of simple random walk and high points of Gaussian free field in two dimensions

Izumi Okada

In this talk, we consider the problems concerning the local time of simple random walk in $\mathbb{Z}^2$. Concreatly, we estimate the exponents as numbers of the favorite sites and the mean numbers of them. These correspond with the results as exponents of late points and of high points of Gaussian free field. We obtain the same value as them for each exponent.

Laws of the Iterated Logarithm for Random Walk on the Random Conductance Model

Chikara Nakamura

In this talk, we will discuss laws of the iterated logarithm (LIL) for random walks on random conductance models (RCM). In the recent progress of random walk on random conductance models, long time Gaussian estimates of heat kernels have been obtained for various random walks on RCMs such as the super critical percolation cluster [1], i.i.d. random conductance models [2], [3], vacant sets of random interlacements and level sets of Gaussian Free Field [5].

We use the heat kernel estimates, and derive LIL and so-called another LIL which describes liminf behavior of random walks. Let $\{X_n^\omega\}_{n\geq 0}$ be the random walk on RCM that enjoys the long time Gaussian heat kernel estimates. Our results are the following;

$$\limsup_{n\to\infty} \frac{d(X_0^\omega, X_n^\omega)}{\sqrt{n \log\log n}} = c_1, \qquad \text{a.s. } P_x^\omega \text{ for all } x, \tag{0.3}$$

$$\liminf_{n\to\infty} \frac{d(X_0^\omega, X_n^\omega)}{\sqrt{n/ \log\log n}} = c_2, \qquad \text{a.s. } P_x^\omega \text{ for all } x. \tag{0.4}$$

where c_1, c_2 are positive constants which may depend on random environments in general. When environments are ergodic, we can take c_1, c_2 as deterministic positive constants. We note that [4] obtained (0.3) for the case of supercritical percolations.

This talk is based on a joint work with T. Kumagai.

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Random walk on Gromov hyperbolic groups: entropy and speed

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Associated to random walks on groups, three basic quantities are defined: entropy, speed and volume growth (exponential growth rate of the group). The fundamental inequality due to Guivarc'h tells that the entropy does not exceed the speed times the volume growth. Vershik (2000) asked about the genuine equality case. We focus on hyperbolic groups, and characterise the equality case; namely, the equality holds if and only if the harmonic measure and a natural measure–a Patterson-Sullivan measure–on the boundary are equivalent. I will compare the result with random walks on the Galton-Watson trees (Lyons-Pemantle-Peres, 1995), and also mention about a random generation algorithm of group elements for a finitely generated (infinite) group due to Vershik. All the notion (such as hyperbolicity) will be explained during the talk.

Elliptic Bessel Process and Elliptic Dyson Model Realized as Temporally Inhomogeneous Processes

Makoto Katori

The Bessel process with the dimensions $D > 1$ and the Dyson model of interacting Brownian motions with the coupling constant $\beta > 0$ are extended to the processes, in which the drift term and the interaction terms are given by the logarithmic derivatives of Jacobi's theta functions. They are called the elliptic Bessel process, $\mathrm{eBES}^{(D)}$, and the elliptic Dyson model, $\mathrm{eDYS}^{(\beta)}$, respectively. Both are realized as temporally inhomogeneous processes defined in a finite time interval. The transformations of them to the quantum Calogero-Moser-Sutherland models with time-dependent potentials lead us to proving that $\mathrm{eBES}^{(D)}$ and $\mathrm{eDYS}^{(\beta)}$ can be constructed as

the time-dependent Girsanov transformations of Brownian motions. In the special cases where $D = 3$ and $\beta = 2$, these processes are represented by the pinned processes with signed measures. We prove that eDYS$^{(2)}$ has the determinantal martingale representation. Then it is proved that eDYS$^{(2)}$ is determinantal for any finite initial configuration, in the sense that all spatio-temporal correlation functions are given by determinants controlled by a single continuous function called the correlation kernel. We will also discuss eDYS$^{(2)}$ with an infinite number of particles.

Infinite Dimensional Stochastic Differential Equations for Dyson's Brownian Motion

Li-Cheng Tsai

Dyson's Brownian Motion (DBM) describes the evolution of the spectra of certain random matrices, and is governed by a system of Stochastic Differential Equation (SDE) with a singular, long-range interaction. In this talk I will discuss the well-posedness of the infinite-dimensional SDE corresponding to the bulk limit of DBM, i.e.

$$X_i(t) = X_i(0) + B_i(t) + \beta \int_0^t \phi_i(\mathbf{X}(s))ds, \ i \in \mathbb{Z}, \tag{0.5}$$

where $\mathbf{X}(s) = (\ldots < X_0(s) < X_1(s) < \ldots)$ describes ordered particles on $\mathbb{R}$, $B_i(t)$, $i \in \mathbb{Z}$, denote independent standard Brownian motions, and the interaction $\phi_i(\mathbf{x})$ takes the form

$$\phi_i(\mathbf{x}) := \frac{1}{2} \lim_{k\to\infty} \sum_{j:|j-i|\leq k} \frac{1}{x_i - x_j},$$

with $\beta \geq 1$ measuring its strength.

Well-posedness of such SDE has been studied extensively for $\beta = 1, 2, 4$, relying on the determinantal or Pfaffian structure, see [2] and the references therein. In this talk I will introduce a different approach [3] based on a monotonicity property. The main result, stated in the following theorem, settles the strong existence and pathwise uniqueness for all $\beta \geq 1$ and for the out-of-equilibrium configuration spaces

$$\mathcal{X}(\alpha, \rho) := \Big\{ (\ldots < x_{-1} < x_0 < x_1 < \ldots) : \sup_{n\in\mathbb{Z}} \big| |x_n - x_0| n^{-1} - \rho \big| n^{\alpha} < \infty \Big\},$$

$$\mathcal{X}^{\mathrm{rg}}(\alpha, \rho, p) := \mathcal{X}(\alpha, \rho) \cap \Big\{ \mathbf{x} : \sup_{m\in\mathbb{Z}_+} \Big(m^{-1} \sum\nolimits_{n\in[-m,m)} |x_n - x_{n+1}|^p \Big) < \infty \Big\},$$

which, in particular, include the lattice configuration $\{x_i\} = \mathbb{Z}$ and the sine process. The idea used here further leads to a finite-to-infinite-dimensional convergence of the SDE. As a corollary, specializing this result at $\beta = 2$, we show that the unique strong solution constructed here exhibits the determinantal structure given by [1].

To state the main result, we define the process-valued analogs of $\mathcal{X}(\alpha,\rho)$ and $\mathcal{X}^{\mathrm{rg}}(\alpha,\rho,p)$ as

$$\mathcal{X}_{\mathcal{T}}(\alpha,\rho) := \Big\{\mathbf{x}(\bullet) \in C([0,\infty))^{\mathbb{Z}} : \sup_{n\in\mathbb{Z}, s\le t} \big||x_n(s) - x_0(s)|n^{-1} - \rho\big| n^{\alpha} < \infty, \forall t < \infty\Big\},$$

$$\mathcal{X}_{\mathcal{T}}^{\mathrm{rg}}(\alpha,\rho,p) := \mathcal{X}_{\mathcal{T}}(\alpha,\rho) \cap \Big\{\mathbf{x}(\bullet) : \sup_{m\in\mathbb{Z}_+, s\le t} \Big(m^{-1} \sum\nolimits_{n\in[-m,m)} |x_n(s) - x_{n+1}(s)|^p\Big) < \infty, \forall t < \infty\Big\}.$$

Theorem ([3]). *Fix any $(\alpha,\rho,p) \in (0,1)\times(0,\infty)\times(1,\infty)$. Given any $\mathbf{x}^{ic} \in \mathcal{X}(\alpha,\rho)$, there exists an $\mathcal{X}_{\mathcal{T}}(\alpha,\rho)$-valued, $\mathscr{F}^{\mathbf{B}}$-adapted solution $\mathbf{X}$ of (0.5) starting from $\mathbf{x}^{ic}$. If, in addition, $\mathbf{x}^{ic} \in \mathcal{X}^{rg}(\alpha,\rho,p)$, this solution $\mathbf{X}$ takes value in $\mathcal{X}_{\mathcal{T}}^{rg}(\alpha,\rho,p)$, and is the unique $\mathcal{X}_{\mathcal{T}}^{rg}(\alpha,\rho,p)$-valued solution in the pathwise sense.*

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Stochastic differential equations related to random matrix theory

Hideki Tanemura

We consider non-colliding Brownian motions with two starting points and two endpoints. When the points are chosen so that the two groups of Brownian motions just touch each other, the scaling limit of the process is the reversible process called the *tacnode process* [1, 2].

Let $m_{\mathrm{tac}}^{2\mathsf{n}}$ be the distribution on the configuration space of n particles given by

$$m_{\mathrm{tac}}^{2\mathsf{n}}(d\boldsymbol{x}_{2\mathsf{n}}) = \frac{1}{Z}\left[\det_{1\le i,j\le 2\mathsf{n}}\Big(\exp\{-\frac{1}{2}|x_i - a_j|^2\}\Big)\right]^2,$$

where $a_j = -\sqrt{\mathsf{n}}$ and $a_{\mathsf{n}+j} = \sqrt{\mathsf{n}}$ for $1 \le j \le \mathsf{n}$. We denote the distribution of $\{\mathsf{n}^{1/6}x_1, \dots, \mathsf{n}^{1/6}x_{2\mathsf{n}}\}$ under $m_{\mathrm{tac}}^{2\mathsf{n}}$ by $\mu_{\mathrm{tac}}^{\mathsf{n}}$. It is proved that $\mu_{\mathrm{tac}}^{\mathsf{n}} \to \mu_{\mathrm{tac}}$, weakly as $\mathsf{n} \to \infty$. The limit distribution μ_{tac} is the reversible measure of the tacnode process, and the determinatal point process with the correlation kernel $K_{\mathrm{tac}}(x, y)$ [2]:

$$K_{\mathrm{tac}}(x, y) \equiv L_{\mathrm{tac}}(x, y) + L_{\mathrm{tac}}(-x, -y), \quad x, y \in \mathbb{R}.$$

where

$$\begin{aligned} L_{\mathrm{tac}}(x, y) &= K_{\mathrm{Ai}}(x, y) + 2^{1/3} \int_{(0,\infty)^2} dudv \ \mathrm{Ai}(y + 2^{1/3}u) R(u, v) \mathrm{Ai}(x + 2^{1/3}v) \\ &\quad -2^{1/3} \int_{(0,\infty)^2} dudv \ \mathrm{Ai}(-y + 2^{1/3}u) \mathrm{Ai}(u + v) \mathrm{Ai}(x + 2^{1/3}v) \\ &-2^{1/3} \int_{(0,\infty)^3} dudvdw \ \mathrm{Ai}(-y + 2^{1/3}u) R(u, v) \mathrm{Ai}(v + w) \mathrm{Ai}(x + 2^{1/3}w). \end{aligned}$$

Here Ai is the Airy function, $K_{\mathrm{Ai}}(x, y) = \int_0^\infty du \ \mathrm{Ai}(\mathrm{x} + \mathrm{u})\mathrm{Ai}(\mathrm{y} + \mathrm{u})$, and $R(x, y)$ is the resolvent operator for the restriction of the Airy kernel to $[0, \infty)$, i.e. the kernel of the operator $R = (I - K_{\mathrm{Ai}})^{-1} K_{\mathrm{Ai}}$ on $L^2[0, \infty)$.

In this talk we study infinite dimensional stochastic differential equations associated with the tacnode process.

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A geometric approach to the Ising model

Hugo Duminil-Copin

In this talk, we will discuss a geometric approach to the Ising model which is based on the so-called random-current representation. We will introduce this representation and review a few results obtained in the past few years, including the sharpness and the continuity of the phase transition.

Estimate on the diameter of a long-range percolation graph on the fractal lattice and its application

Jun Misumi

In this talk, we discuss the long-range percolation model on the finite-stage subgraph of the pre-Sierpinski gasket, which is one of the fundamental fractal lattices. Especially, we give several bounds for the diameter of the random graph obtained by such a model. The corresponding problem has been well studied on the long-range percolation on $\{0, 1, 2, \cdots, n\}^d \subset \mathbb{Z}^d$ ($d = 1, 2, 3, \cdots$), and the result of this talk is a natural analogy of it in a certain sense. Further, we also present some estimates on the mixing time of the random walk on such a random graph.

Enlargement of subgraphs of infinite graphs by Bernoulli percolation

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We consider changes of properties of subgraphs of an infinite graph if we enlarge the subgraphs by adding Bernoulli percolation on the infinite graph to the subgraphs. We give a pair of an infinite graph, a subgraph of it, and, a property for graphs. Then we can define two new “critical probabilities” of the pair, in a manner similar to the way that the ordinal critical probability is defined. We focus on the cases that a property is being a transient subgraph, having finitely many or no cut points, being a recurrent subset, and, being connected. We compare the two new critical probabilities with the ordinal critical probability. Our results depend heavily on a choice of a pair of an infinite graph, a subgraph of it, and, a property.

Concentration of First passage percolation

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First passage percolation (FPP) was first introduced by Hammersley and Welsh in 1965. It can be thought of as a model for the speed to percolate some material. In this talk, we introduce a new family of FPP models and investigate the scaling limits of minimum passage time. We shall give precise definitions below.

The random environment is modelled by independent and identically distributed Bernoulli random variables $(\{\eta(j,x)\}_{(j,x)\in\mathbb{N}\times\mathbb{Z}^d}, Q)$ with parameter p;

$$Q(\eta(0,0)=1)=p\in(0,1).$$

And we define a configration ω_p by

$$\omega_p=\sum_{(k,x)\in\mathbb{N}\times\mathbb{Z}^d}(1-\eta(k,x))\delta_{(k,p^{1/d}x)}.$$

And besides, let (Q,ω_0) be the Poisson point process on $\mathbb{N}\times\mathbb{R}^d$ whose intensity is the product of the counting measure and Lebesgue measure. With some abuse of notation we will frequently identify ω_p, and more generally any point measure, with its support. Since the scaling factor $p^{1/d}$ works, we realize that ω_p converges to ω_0 as $p\uparrow 0$. Given a realization of ω_p, we define the minimum passage time from 0 to n by

$$T_n(\omega_p)=\min\left\{\sum_{k=1}^n|x_{k-1}-x_k|^\alpha : x_0=0 \text{ and } \{(k,x_k)\}_{k=1}^n\subset\omega_p\right\}. \tag{0.6}$$

Now, a direct application of the subadditive ergodic theorem shows that the limit

$$\mu_p=\lim_{n\to\infty}\frac{1}{n}T_n(\omega_p) \tag{0.7}$$

exists Q-almost surely and equals to $\lim\frac{1}{n}Q[T_n(\omega_p)]$. The limit μ_p, so-called time constant, is non random. Observe also that definition (0.6) makes perfect sense when $p=0$, yielding a limit μ_0 in (0.7).

Our purpose is to establish the concentration results for the minimum passage time T_n. As for $\alpha>1$, concentration results are closely related to the maximum jump of optimal path, where optimal path is a path which attains the minimum passage time. Hence, the maximum jump of optimal path is key quantity in my talk.

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Concentrations for the simple random walk in unbounded nonnegative potentials

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We consider the simple random walk in i.i.d. nonnegative potentials on the d-dimensional cubic lattice $\mathbb{Z}^d$ ($d \geq 2$). Let $(S_k)_{k=0}^{\infty}$ be the simple random walk on $\mathbb{Z}^d$ starting at 0, and write P^0 for its law. In addition, $\omega = (\omega(z))_{z \in \mathbb{Z}^d}$ are i.i.d. nonnegative random variables, which are called potentials (let $\mathbb{P}$ be the law of potentials and $\mathbb{E}$ its expectation). Then, the main object in this talk is the cost of traveling from 0 to x for the random walk in the potential, i.e.,

$$a(0,x) := -\log E^0\left[\exp\left\{-\sum_{k=0}^{H(x)-1} \omega(S_k)\right\}\mathbf{1}\{H(x) < \infty\}\right],$$

where E^0 is the expectation with respect to P^0, and $H(x)$ is the first passage time through x.

We now introduce the following assumptions for the potential:

(A1) $\mathbb{E}[e^{\gamma\omega(0)}] < \infty$ for some $\gamma > 0$.

(A2) $\mathbb{E}[\omega(0)^2] < \infty$.

(A3) The law of $\omega(0)$ has strictly positive support.

Under these assumptions, the following are main results, i.e., we have the exponential concentration for the upper tail, and the Gaussian concentration for the lower tail.

Theorem. *Assume (A1). In addition, suppose that (A3) is valid if $d = 2$. Then, there exist constants $0 < C_1, C_2, C_3 < \infty$ such that for all large $x \in \mathbb{Z}^d$ and for all $t \leq C_1\|x\|_1$,*

$$\mathbb{P}\big(a(0,x) - \mathbb{E}[a(0,x)] \geq t\|x\|_1^{1/2}\big) \leq C_2 e^{-C_3 t}.$$

Theorem. *Assume (A2). In addition, suppose that (A3) is valid if* $d = 2$*. Then, there exists a constant* $0 < C_4 < \infty$ *such that for all large* $x \in \mathbb{Z}^d$ *and for all* $t \geq 0$*,*

$$\mathbb{P}\big(a(0,x) - \mathbb{E}[a(0,x)] \leq -t\|x\|_1^{1/2}\big) \leq e^{-C_4 t^2}.$$

In the case where potentials have bounded and strictly positive support, these concentrations have already proved by Ioffe–Velenik [1], Sodin [2], Sznitman [3]. Theorems and extend those results to unbounded and nonengative potentials.

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Ginibre interaction Brownian motions are sub-diffusive

Hirofumi Osada

Ginibre interacting Brownian motions are infinitely many particles moving in the plane. They interact through the logarithmic potential with inverse temperature $\beta = 2$. The equilibrium state of the associated unlabeled dynamics is Ginibre point process, which is a translation and rotation invariant determinantal point process given by the exponential kernel. In this talk, I prove that each tagged particle of Ginibre interacting Brownian motions is sub-diffusive when the total system starts from the Ginibre point process. This results shows that the geometric rigidity of the Ginibre point process is inherited to the Ginibre interacting Brownian motions. Interacting Brownian motions with Ruelle's class potentials and simple exclusion processes are always diffusive in Euclidean spaces with dimensions greater than 1. One dimension is very special, because it has a topological constrain which suppress the motion of the tagged particles. In fact, it is known that the tagged particles with no collisions are sub-diffusive because particles cannot change the order of their positions. In dimension 2, there exists no such a topological constrain. We see that the strength of the logarithmic potential yields strikingly novel phenomena for the associated stochastic dynamics.

Largest eigenvalues in random matrix beta-ensembles: identification of the limit

Vadim Gorin

The scaling limits of largest eigenvalues in random matrix ensembles can be studied by a number of methods including determinantal point processes, Dyson Brownian Motion, moments method, tridiagonal matrices. However, the answers produced by different methods are hard to match with each other. In my talk I will discuss an interplay between the approaches. The outcomes include a novel scaling limit for the differences between largest eigenvalues in submatrices and a Feynman-Kac type formula for the semigroup spanned by the Stochastic Airy Operator.

On the probability that Laplacian interface models stay positive in subcritical dimensions

Hironobu Sakagawa

We consider a class of effective interface models on $\mathbb{Z}^d$ which is known as a model of semiflexible membrane. The interaction depends on discrete Laplacian and the field displays huge fluctuations when $d \leq 3$. We give an estimate of the probability that the field stays positive and its behaviors differ greatly from those of the higher dimensional case or effective interface models with gradient interactions.

On the topology of random simplicial complexes

Tomoyuki Shirai

Random simplicial complexes naturally arise from point processes as Čech complex or Rips-Vietoris complex. Some topological properties of point processes are refleceted in these complexes. One of the well-developed tools for analyzing such simplicial complexes is persistent homology theory, which gives us information about appearance and disappearance of homology classes as persistence diagram when we are given an increasing sequence of simplicial complexes as input. In this talk, we discuss persistent homology for certain random simplicial complexes and some related topics.

This talk is based on a joint work with Yasuaki Hiraoka (AIMR, Tohoku Univ.)

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Quenched localisation in the Bouchaud trap model with slowly varying traps

David Croydon

I will discuss a recent joint work with Stephen Muirhead (University College London), in which we study localisation properties of the Bouchaud trap model on the positive integers in the case that the trap distribution has a slowly varying tail at infinity. (The Bouchaud trap model evolves as a continuous time simple symmetric random walk, but with holding times whose mean is selected from the trap distribution independently for each site.) In particular, I will describe that for each $N = 2, 3, 4, \dots$ there exists a slowly varying tail such that the model localises on exactly N sites. Key intuition for this result is provided by an observation about the sum-max ratio for sequences of independent and identically distributed random variables with a slowly varying distributional tail.

Infinite particle systems of long range jumps with long range interactions

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We present general theorem of constructing infinite particle systems of jump type with long range interaction by the Dirichlet form technique. Our theorem includes the case that each particles undergoes α-stable processes and interaction between particles given by the logarithmic potential appearing random matrix theory and potentials of Ruelle's class with polynomial decay. In the assumption a jump rate is restricted by the 1-correlation function of the equilibrium measure. This is necessary for infinitely particles not to concentrate on any compact set. In particular this assumption is satisfied for any $\alpha \in (0, 2)$ in case a equilibrium measure is translation invariant.

Finite particle approximation of infinite-dimensional SDE related to random matrices

Yosuke Kawamoto

We consider the interacting Brownian motions with infinitely many particles. This system is described by infinite-dimensional SDE (ISDE). We construct the general theory of finite particle approximation, that is, the solution of finite-dimensional SDE converges to the solution of corresponding ISDE. This theory can be applied to ISDE interacted with Ruelle's class potentials, for example, Riesz potential, Lennard-Jones 6-12 potential. Furthermore, we prove finite particle approximation of Dyson's Brownian motion and ISDE related to Airy random point field by using this general theory. These dynamics are interacting Brownian motions with logarithmic potential, which is long range potential, and closely related to random matrices. This talk is based on joint work with H. Osada.